MATHEMATICS

Grade 7

Book 1

CAPS

Learner Book

Developed and funded as an ongoing project by the Sasol Inzalo Foundation in partnership with the Ukuqonda Institute.

Published by The Ukuqonda Institute
9 Neale Street, Rietondale 0084
Registered as a Title 21 company, registration number 2006/026363/08
Public Benefit Organisation, PBO Nr. 930035134
Website: http://www.ukuqonda.org.za

First published in 2014

ISBN: 978-1-920705-24-4

This book was developed with the participation of the Department of Basic Education of South Africa with funding from the Sasol Inzalo Foundation.

Contributors:
Piet Human, Erna Lampen, Marthinus de Jager, Louise Keegan, Paul van Koersveld, Nathi Makae, Enoch Masemola, Therine van Niekerk, Alwyn Olivier, Cerenus Pfeiffer, Renate Röhrs, Dirk Wessels, Herholdt Bezuidenhout

Illustrations and computer graphics:
Leonora van Staden, Lisa Steyn Illustration
Zhandré Stark, Lebone Publishing Services
Computer graphics for chapter frontispieces: Piet Human

Cover illustration: Leonora van Staden

Text design: Mike Schramm

Layout and typesetting: Lebone Publishing Services

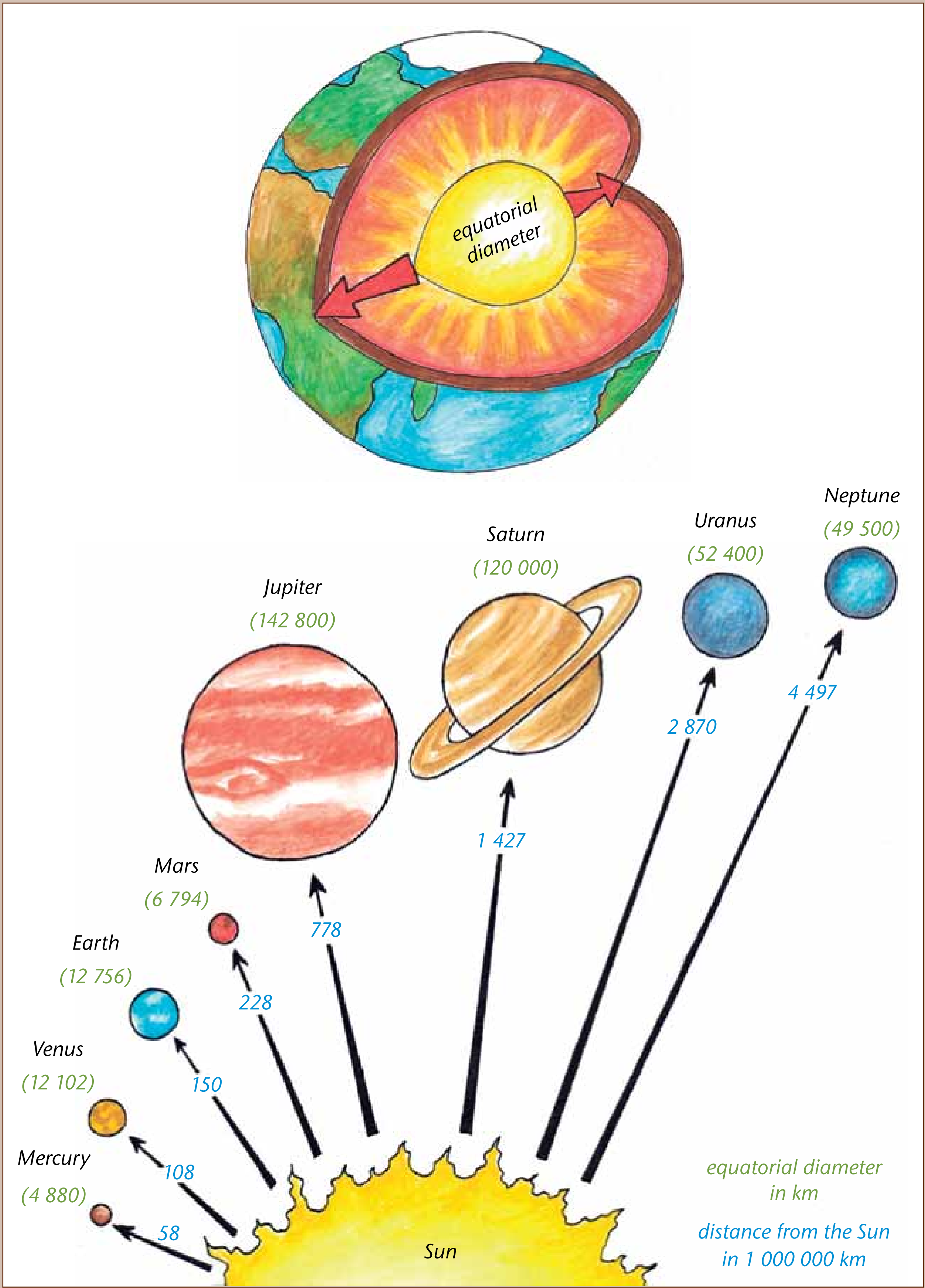
Printed by: [printer name and address]

Table of contents

Term 2

Chapter 1
Working with whole numbers

In this chapter you will learn more about whole numbers. You will learn about different ways to express whole numbers as sums and products. You will learn about different ways of doing calculations, and different ways of recording your work when doing calculations. You will strengthen your skills to do calculations and to solve problems.


equatorial diameter
Neptune
(49 500)
Uranus
(52 400)
Saturn
(120 000)
Jupiter
(142 800)
Mars
(6 794)
Earth
(12 756)
Venus
(12 102)
Mercury
(4 880)
4 497
2 870
1 427
778
228
150
108
58
Sun
equatorial diameter in km
distance from the Sun in 1 000 000 km

1 Working with whole numbers

1.1 Revision

Do not use a calculator at all in section 1.1.

BUILD NUMBERS UP AND BREAK NUMBERS DOWN

1. Write each of the following sums as a single number:
   (a) 4 000 + 800 + 60 + 5 ....
   (b) 8 000 + 300 + 7 ....
   (c) 40 000 + 9 000 + 200 + 3 ....
   (d) 800 000 + 70 000 + 3 000 + 900 + 2 ....
   (e) 8 thousands + 7 hundreds + 8 units ....
   (f) 4 hundred thousands + 8 ten thousands + 4 hundreds + 9 tens ....

The word **sum** is used to indicate two or more numbers that have to be added.

The answer obtained when the numbers are added, is also called the **sum**. We say: 20 is the sum of 15 and 5.

2. What is the sum of 8 000 and 24? ....

3. Write each of the numbers below as a sum of units, tens, hundreds, thousands, ten thousands and hundred thousands, like the numbers were given in question 1(e) and (f).

When a number is written as a sum of units, tens, hundreds, thousands etc., it is called the **expanded notation**.

   (a) 8 706 = ...
   (b) 449 203 = ...
   ....
   (c) 83 490 = ...
   ....
   (d) 873 092 = ...
   ....

4. Arrange the numbers in question 3 from smallest to biggest.

   ....

5. Write the numbers in expanded notation (for example, 791 = 700 + 90 + 1).
   (a) 493 020 ....
   (b) 409 302 ....
   (c) 490 032 ....
   (d) 400 932 ....

6. Arrange the numbers in question 5 from biggest to smallest.

....

7. Write each sum as a single number.
   (a) 600 000 + 40 000 + 27 000 + 100 + 20 + 34 ....
   (b) 320 000 + 40 000 + 8 000 + 670 + 10 + 5 ....
   (c) 500 000 + 280 000 + 7 000 + 300 + 170 + 38 ....
   (d) 4 hundred thousands + 18 ten thousands + 4 hundreds + 29 tens + 5 units

   ....

8. Write each sum as a single number.
   (a) 300 000 + 70 000 + 6 000 + 400 + 80 + 6 ....
   (b) 400 000 + 20 000 + 2 000 + 500 + 10 + 3 ....
   (c) 500 000 + 40 000 + 7 000 + 300 + 60 + 6 ....
   (d) 800 000 + 90 000 + 7 000 + 800 + 90 + 8 ....
   (e) 300 000 + 110 000 + 12 000 + 400 + 110 + 3 ....

9. In each case, add the two numbers. Write the answer in expanded form and also as a single number.
   (a) The number in 8(a) and the number in 8(b)

   ....

   (b) The number in 8(c) and the number in 8(b)

   ....

   (c) The number in 8(c) and the number in 8(a)

   ....

   (d) The number in 8(d) and the number in 8(a)

   ....

10. (a) Subtract the number in 8(b) from the number in 8(d).

   ....

   (b) Are the numbers in 8(b) and 8(e) the same? ....
   (c) Subtract the number in 8(a) from the number in 8(b).

   ....

11. Write each of the following products as a single number:

(a) 2×3 ....

(b) $2 \times 3 \times 5$ ....

(c) $2 \times 3 \times 5 \times 7$ ....

(d) $2 \times 3 \times 5 \times 7 \times 2$

....

(e) $2 \times 3 \times 5 \times 7 \times 2 \times 2$

....

The word **product** is used to indicate two or more numbers that have to be multiplied.

The answer obtained when numbers are multiplied, is also called the **product**. We say: 20 is the product of 2 and 10.

12. (a) What is the product of 20 and 500? ....

(b) Write 1 000 as a product of 5 and another number. ....

(c) Write 1 000 as a product of 50 and another number. ....

(d) Write 1 000 as a product of 25 and another number. ....

(e) What is the product of 2 500 and 4? ....

(f) What is the product of 250 and 40? ....

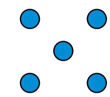
13. In the table on the right, the number in each yellow cell is formed by adding the number in the red row above it to the number in the blue column to its left. Write the correct numbers in all the empty yellow cells.

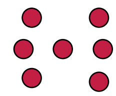
+	2	3	4	5
10				
20				
30			34	
40				
50				
60		63		65
70				

14. The table below is formed in the same way as the table on the right. Fill in all the cells for which you know the answers immediately. Leave the other cells open for now.

+	8	5	4	9	7	3	6	18	36	57
7										64
3						6				
9										
5										
8										
6										
4										

MULTIPLES

1. In the arrangement below, the blue dots are in groups like this: 

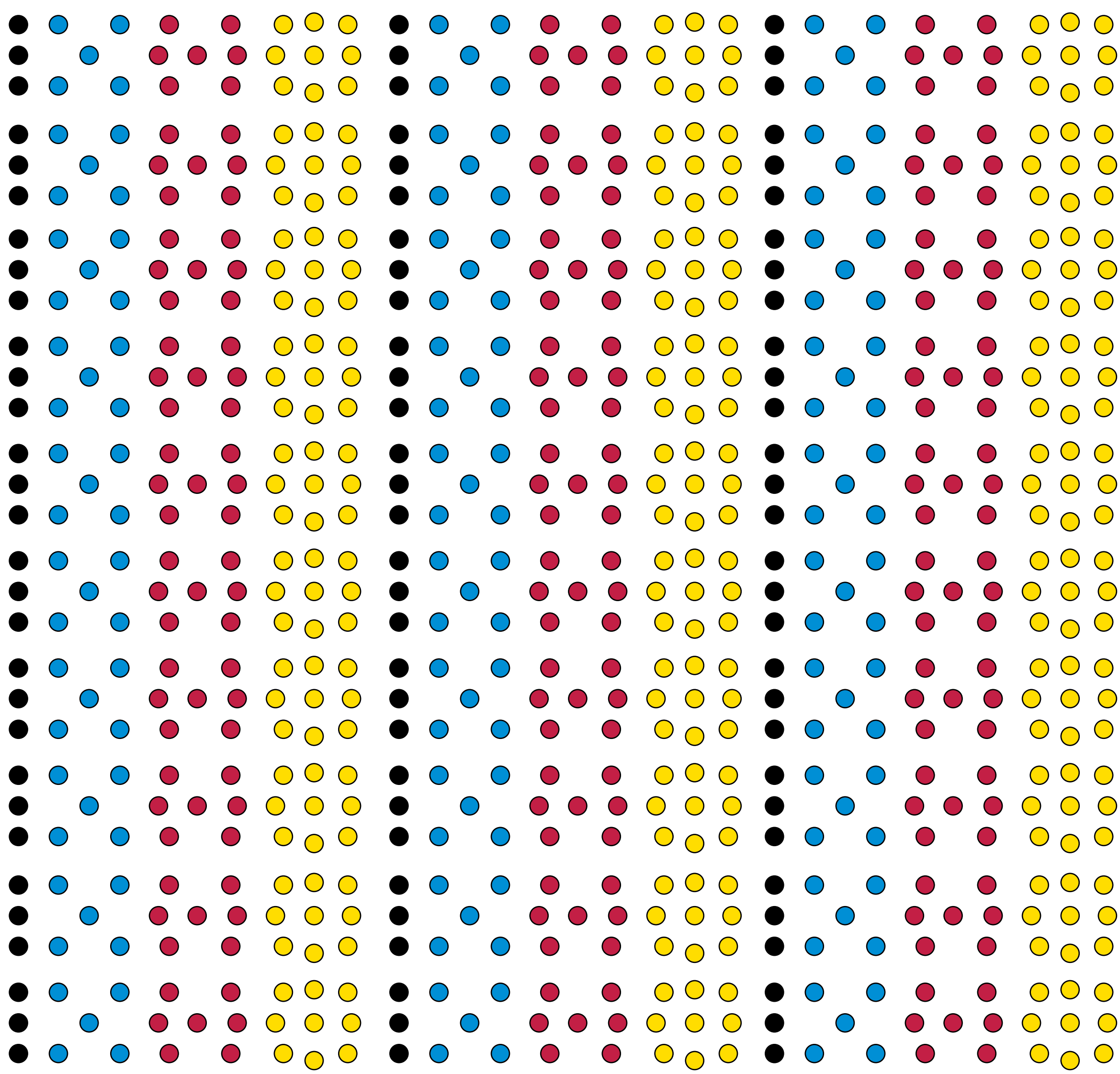
   The red dots are in groups like this: 

   (a) How would you go about finding the number of blue dots below, if you do not want to count them one by one?

   ....

   (b) Implement your plan, to find out how many blue dots there are.

   ....



Suppose you want to know how many black dots there are in the arrangement on page 6. One way is to **count in groups** of three. When you do this, you may have to point with your finger or pencil to keep track.

The counting will go like this: *three*, *six*, *nine*, *twelve*, *fifteen*, *eighteen* . . .

Another way to find out how many black dots there are is to **analyse** the arrangement and **do some calculations**. In the arrangement, there are ten rows of threes from the top to the bottom, and three columns of threes from left to right, just as in the table alongside.

3	3	3
3	3	3
3	3	3
3	3	3
3	3	3
3	3	3
3	3	3
3	3	3
3	3	3
3	3	3

One way to calculate the total number of black dots is to do $3 \times 10 = 30$ for the dots in each column, and then $30 + 30 + 30 = 90$. Another way is to add up in each row ($3 + 3 + 3 = 9$) and then multiply by 10: $10 \times 9 = 90$. A third way is to notice that there are $3 \times 10 = 30$ groups of three, so the total is $3 \times 30 = 90$.

2. When you determined the number of blue dots in question 1(b), did you count in fives, or did you analyse and calculate, or did you use some other method? Now use a different method to determine the number of blue dots and check whether you get the same answer as before. Describe the method that you now use.

   ....

3. The numbers that you get when you count in fives are called **multiples** of five. Draw circles around all the multiples of 5 in the table below.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100
101	102	103	104	105	106	107	108	109	110
111	112	113	114	115	116	117	118	119	120

4. How many red dots are there in the arrangement on page 6? Describe the method that you use to find this out.

....

....

5. (a) Underline all the multiples of 7 in the table in question 3.
   (b) Which multiples of 5 in the table are also multiples of 7?

A number that is a multiple of 5, and also a multiple of 7, is called a **common multiple** of 5 and 7.

....

6. How many yellow dots are there in the arrangement on page 6? Describe the method that you use to find this out.

....

....

7. (a) Cross out all the multiples of 9 in the table in question 3.
   (b) Which numbers in the table in question 3 are common multiples of 7 and 9?

....

8. (a) Look at the numbers in the yellow cells of the table below. How are these numbers formed from the numbers in the red row and the numbers in the blue column?

....

   (b) Fill in all the cells for which you know the answers immediately. Leave the other cells open for now.

×	8	5	4	9	7	3	6	2	10	20
7										
3										
9									90	
5			20							
8										
6										
4										
2										
10		50								
20						60				

9. Write down the first thirteen multiples of each of the numbers in the column on the left. The multiples of 4 are already written in, as an example.

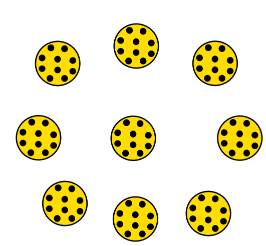
1	2	3	4	5	6	7	8	9	10	11	12	13
2												
3												
4	8	12	16	20	24	28	32	36	40	44	48	52
5												
6												
7												
8												
9												
10												
11												
12												
13												

10. Complete this table. For some cells, you may find your table of multiples above helpful.

×	6	5	7	9	4	5	3	8	10	50
8										
6										
7										
9										
5										
3										
4										
2										

11. Go back to the table in question 8(b). If you can easily fill in the numbers in some of the open spaces now, do it.

12. Suppose there are 10 small black spots on each of the yellow dots in the arrangement on page 6. How many small black spots would there be on all the yellow dots together, in the arrangement on page 6?



....

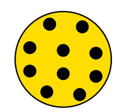
MULTIPLES OF 10, 100, 1 000 AND 10 000

1. How many spotted yellow dots are there on page 11? Explain what you did to find out.

2. How many learners are there in your class?

   Suppose each learner in the class has a book like this. How many spotted yellow dots are there on the same page (that is, on page 11) of all these books together?

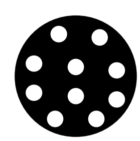
3. Each yellow dot has 10 small black spots, as you can see on this enlarged picture.

   (a) How many small black spots are there on page 11?

   (b) How many small black spots are there on page 11 in all the books in your class?

4. Here is a very big enlargement of one of the black spots on the yellow dots. There are 10 very small white spots on each small black spot. How many very small white spots are there on all the black spots on page 11 together?

5. (a) How many very small white spots are there on 10 pages like page 11?

   (b) How many very small white spots are there on 100 pages like page 11?

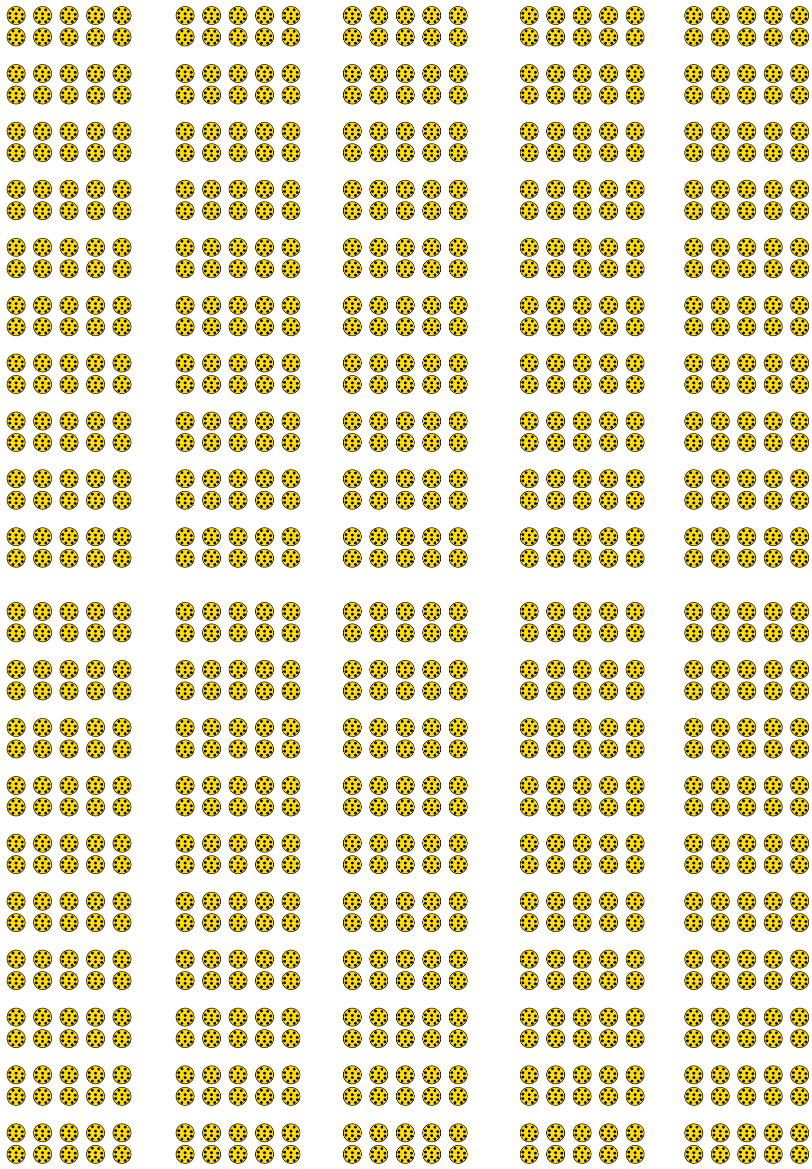
10 tens are a **hundred**: $10 \times 10 = 100$

10 hundreds are a **thousand**: $10 \times 100 = 1\ 000$

10 thousands are a **ten thousand**: $10 \times 1\ 000 = 10\ 000$

10 ten thousands are a **hundred thousand**: $10 \times 10\ 000 = 100\ 000$

10 hundred thousands are a **million**: $10 \times 100\ 000 = 1\ 000\ 000$



6. (a) Write 7 000 + 600 + 80 + 4 as a single number. ....

   (b) Write 10 times the number in (a) in expanded notation and as a single number.

   ....

   (c) Write 100 times the number in (a) in expanded notation and as a single number.

   ....

7. Write each of the following numbers in expanded notation:

   (a) 746 ....

   (b) 7 460 ....

   (c) 74 600 ....

   (d) 746 000 ....

   (e) 7 460 000 ....

8. (a) Write 10 000 as a product of 10 and one other number. ....

(b) Write 10 000 as a product of 100 and one other number. ....

(c) Write 100 000 as a product of 10 and one other number. ....

(d) Write 100 000 as a product of 1 000 and one other number. ....

(e) Write 1 000 000 as a product of 1 000 and one other number. ....

9. In the table below, fill in all the cells for which you know the answers immediately. Leave the other cells open for now.

×	10	20	30	40	50	60	70	80	90	100
2										
3										
4										
5										
6										
7										
8										
9										
10										
11										
12										

10. Fill in all the cells in the table for which you know the answers immediately. Leave the other cells open for now.

×	100	200	300	400	500	600
2						
3						
4						
5						
6						
7						
8						
9						
10						
11						
12						

11. How many multiples of 10 are smaller than 250? You may make an estimate, and then write the multiples down to check.
    (a) Estimate: ....
    (b) Check: ....

....

12. In each case first estimate, then check by writing all the multiples down and counting them.
    (a) How many multiples of 100 are smaller than 2 500?

....

....

....

    (b) How many multiples of 250 are smaller than 2 500?

....

....

    (c) How many numbers smaller than 2 500 are multiples of both 100 and 250?

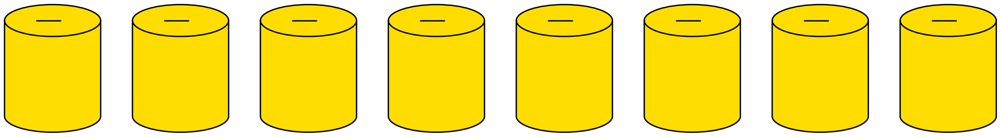
....

....

    (d) How many numbers smaller than 2 500 are multiples of both 250 and 400?

....

13. In each of the tins below, there are three R10 notes, three R20 notes, three R100 notes and three R200 notes.



Zain wants to know what the total value of all the R10 notes in all the tins is. He decides to find this out by counting in 30s, so he says: *thirty*, *sixty*, *ninety* . . . and so on while he points at one tin after another.

    (a) Complete what Zain started to do. ....

....

    (b) Count in 300s to find out what the total value of all the R100 notes in all the tins is.

....

14. (a) How much money is there in total in the eight yellow tins in question 13?

....

....

....

(b) Join with two classmates and tell them how you worked to find the total amount of money.

15. (a) Investigate what is easiest for you, to count in twenties or in thirties or in fifties, up to 500.

(b) Many people find it easier to count in fifties than in thirties. Why do you think this is so?

....

....

....

16. What do you expect to be the most difficult, to count in forties or in seventies or in nineties? Investigate this and write a short report.

....

....

....

....

Here is some advice that can make it easier to count in certain counting units, for example in seventies.

It feels easier to count in fifties than in seventies because you get to multiples of 100 at every second step:

fifty, **hundred**, one hundred and fifty, **two hundred**, two hundred and fifty, **300**, 350, **400**, 450, **500** ... and so on.

When you count in seventies, this does not happen:

seventy, one hundred and forty, two hundred and ten, two hundred and eighty ...

It may help you to cross over the multiples of 100 in two steps each time, like this:

70 + *30* → **100** + *40* → **140** + *60* → **200** + *10* → **210** + *70* → **280** ...

30 + 40 = 70 *60 + 10 = 70*

In this way, you make the multiples of 100 act as "stepping stones" for your counting.

17. (a) Count in forties up to 1 000. Try to use multiples of 100 as stepping stones. You can write the numbers below while you count.

....

....

(b) Write down the first twenty multiples of 80.

....

....

(c) Write down the first twenty multiples of 90.

....

....

(d) Write down the first ten multiples of 700.

....

....

18. Complete this table.

×	60	50	70	90	40	20	30	80
8								
6								
7								
9								
5								
3								
4								
2								
70								
30								
60								
80								
40								
90								
50								
20								

DOUBLING AND HALVING

1. Write the next eight numbers in each pattern:
   (a) 1 2 4 8 16 32 ....
   (b) 3 6 12 24 ....
   (c) 5 10 20 40 ....
   (d) 5 10 15 20 ....
   (e) 6 12 24 48 ....

2. Which pattern or patterns in question 1 are *not* formed by **repeated doubling**?

   ....

The pattern 3 6 12 24 48 ... may be called the **repeated doubling pattern** that starts with 3.

3. Write the first nine terms of the repeated doubling patterns that start with the numbers in the left column of the table. The pattern for 13 has been completed as an example.

2								
3								
4								
5								
6								
7								
8								
9								
10								
11								
12								
13	26	52	104	208	416	832	1 664	3 328
14								
15								
16								
17								
18								
19								

Doubling can be used to do multiplication.

For example, 29×8 can be calculated as follows:

8 doubled is 16, so $16 = 2 \times 8$	(step 1)
16 doubled is 32, so $32 = 4 \times 8$	(step 2)
32 doubled is 64, so $64 = 8 \times 8$	(step 3)
64 doubled is 128, so $128 = 16 \times 8$	(step 4). Doubling again will go past 29×8.

$16 \times 8 + 8 \times 8 + 4 \times 8 = (16 + 8 + 4) \times 8 = 28 \times 8$.

So $28 \times 8 = 128 + 64 + 32$ which is 224. So $29 \times 8 = 224 + 8 = 232$.

4. Work as in the above example to calculate each of the following. Write only what you need to write.

   (a) 37×21

   (b) 17×41

   ....

   ....

5. Continue each repeated halving pattern as far as you can:

   (a) 1 024 512 256 128 ....

   (b) 64 000 32 000 16 000 8 000 ....

Halving can also be used to do multiplication.

For example, 37×28 can be calculated as follows:

$100 \times 28 = 2\,800$. Half of that is 50×28 which is half of 2 800, that is 1 400.

Half of 50×28 is half of 1 400, so 25×28 is 700.

$10 \times 28 = 280$, so $25 \times 28 + 10 \times 28 = 980$, so $35 \times 28 = 980$.

$2 \times 28 = 2 \times 25 + 2 \times 3 = 56$, so 37×28 is $980 + 56 = 1\,036$.

6. $80 \times 78 = 6\,240$. Use this information to work out each of the following:

   (a) 20×78

   (b) 37×78

   ....

   ....

   ....

If chickens cost R27 each, how many chickens can you buy with R2 400? A way to use halving to work this out is shown on the next page.

100 chickens cost 100 × 27 = R2 700. That is more than R2 400. 50 chickens cost half as much, that is R1 350.

So I can buy 50 chickens and even more.

Half of 50 is 25 and half of R1 350 is R675.

So 75 chickens cost R1 350 + R675, which is R2 025. So there is R375 left.

10 chickens cost R270, so 85 chickens cost R2 025 + R270 = R2 295. There is R105 left.

3 chickens cost 3 × R25 + 3 × R2 = R81.

I can buy 88 chickens and that will cost R2 376.

	Total cost	Thinking
100	R2 700	
50	R1 350	half of R2 700
25	R675	half of R1 350
75	R2 025	50 + 25 chickens
10	R270	10 × R27
85	R2 295	75 + 10 chickens
3	R81	3 × R27
88	R2 376	85 + 3 chickens

7. Use halving as in the above example to work out how many books at R67 each a school can buy with R5 000.

   ....

   ....

	Total cost	Thinking

USING MULTIPLICATION TO DO DIVISION

1. R7 500 must be shared between 27 netball players. The money is in R10 notes, and no small change is available.
   (a) How much money will be used to give each player R100? ....
   (b) Do you think there is enough money to give each player R200? ....
   (c) Do you think there is enough money to give each player R300?

   ....

   (d) How much of the R7 500 will be left over, if each player is given R200?

   ....

   (e) Is there enough money left to give each player R50 more, in other words a total of R250 each?

   ....

   (f) What is the highest amount that can be given to each player, so that less than R270 is left over? Remember that you cannot split up the R10 notes.

   ....

2. Work like you did in question 1 to solve this problem:

   There is 4 580 m of string on a big roll. How many pieces of 17 m each can be cut from this roll?

   Hint: You may start by asking yourself how much string will be used if you cut off 100 pieces of 17 m each.

   ....

   ....

   ....

   ....

   ....

   ....

   ....

3. Work like you did in questions 1 and 2 to solve this problem:

   A shop owner has R1 800 available with which he can buy chickens from a farmer. The farmer wants R26 for each chicken. How many chickens can the shop owner buy?

   ....

   ....

   ....

   ....

   ....

   ....

What you actually did in questions 1, 2 and 3 was to calculate 7 500 ÷ 27, 4 580 ÷ 17 and 1 800 ÷ 26. You solved division problems. Yet most of the work was to do multiplication, and a little bit of subtraction.

When you had to calculate 1 800 ÷ 26 in question 3, you may have asked yourself:

With what must I multiply 26, to get as close to 1 800 as possible?

Division is called the **inverse** of multiplication.

Multiplication is called the **inverse** of division.

Multiplication and division are **inverse operations**.

1.2 Ordering and comparing whole numbers

HOW FAR CAN YOU COUNT, AND HOW FAR IS FAR?

1. How long will it take to count to a million? Let us say it takes one second to count each number. Find out how long is one million seconds. Work in your exercise book. Give your final answer in days, hours and seconds.

2. Write 234 500 320 in words.

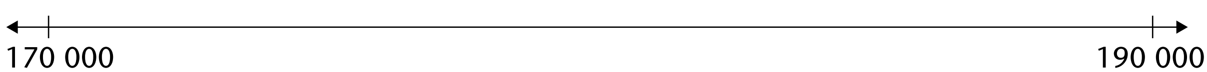
3. In each case insert one of the symbols > or < to indicate which number is the smaller of the two.

   (a) 876 243 876 234 (b) 534 616 543 016

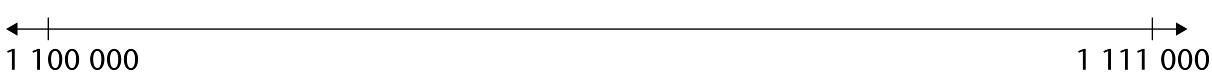
   (c) 701 021 698 769 (d) 103 232 99 878

4. In each case place the numbers on the number line as carefully as you can.

   (a) 185 000; 178 000; 170 900; 180 500



   (b) 1 110 000; 1 102 900; 1 100 500; 1 105 050



The first row in the table shows the average distances of the planets from the Sun. These distances are given in **millions of kilometres**.

One million kilometres is **1 000 000 km**.

The distances from the Sun are called average distances, because the planets are not always the same distance from the Sun. Their orbits are not circles.

Planet	Mercury	Venus	Earth	Mars	Jupiter	Saturn	Uranus	Neptune
Distance from the Sun	58 million km	108 million km	150 million km	228 million km	778 million km	1 427 million km	2 870 million km	4 497 million km
Equatorial diameter	4 880 km	12 102 km	12 756 km	6 794 km	142 800 km	120 000 km	52 400 km	49 500 km

The information in the table is also given in the drawings on page 2. Study the top drawing to find out what equatorial diameter means.

5. Which planet is the second farthest planet from the Sun? ....

6. How does Mars' distance from the Sun compare to that of Venus? Give two possible answers.

   ....

   ....

7. Arrange the planets from the smallest to the biggest.

   ....

Sometimes we do not need to know the exact number or exact amount. We say a loaf of bread costs about R10, or a bag of mealie meal costs about R20. The loaf of bread may cost R8 or R12 but it is close to R10. The mealie meal may cost R18 or R21 but it is close to R20.

When you read in a newspaper that there were 15 000 spectators at a soccer game, you know that that is not the actual number. In the language of mathematics we call this process **rounding off** or **rounding**.

ROUNDING TO 5s, 10s, 100s AND 1 000s

> To round off to the **nearest 5**, we round numbers that end in 1 or 2, or 6 or 7 **down** to the closest multiple of 5. We round numbers that end in 3 or 4, or 8 or 9 **up** to the closest multiple of 5.

For example, 233 is rounded down to 230, 234 is rounded up to 235, 237 is rounded down to 235 and 238 is rounded up to 240.

1. Round the following numbers to the nearest 5 by checking the **unit value**:

   (a) 612 (b) 87 (c) 454 (d) 1 328

   ....

> To round off to the **nearest 10**, we round numbers that end in 1, 2, 3 or 4 **down** to the closest multiple of 10 (or decade). We round numbers that end in 5, 6, 7, 8 or 9 **up** to the closest multiple of 10.

For example, if you want to round off 534 to the nearest 10, you have to look at the units digit. The units digit is 4 and it is closer to 0 than to 10. The rounded off number will be 530.

2. Round the following numbers to the nearest 10 by checking the **unit value**:

   (a) 12 .... (b) 87 .... (c) 454 .... (d) 1 325 ....

When **rounding to the nearest 100**, we look at the last **two digits** of the number. If the number is less than 50 we **round down** to the lower 100. If the number is 50 or more we **round up** to the higher 100.

3. Complete the table.

	Round to the nearest 5	Round to the nearest 10	Round to the nearest 100
681			
5 639			
5 361			
12 458			

When **rounding to the nearest 1 000**, we look at the hundreds. Is the hundreds value less than, equal to or greater than 500? If less than 500, round down (the thousands value stays the same), if equal to 500 round up, and if greater than 500 round up too.

When **rounding to the nearest 10 000**, we look at the thousands. Is the thousands value less than, equal to or greater than 5 000? If less than 5 000, round down (the ten thousands value stays the same), if equal to 5 000 or greater than 5 000 round up.

4. Complete the table.

	Round to the nearest 1 000	Round to the nearest 10 000
142 389		
343 621		
356 552		
100 489		

1.3 Factors, prime numbers and common multiples

DIFFERENT WAYS TO PRODUCE THE SAME NUMBER

The number 80 can be produced by multiplying 4 and 20: $4 \times 20 = 80$.
The number 80 can also be produced by multiplying 5 and 16.

1. In what other ways can 80 be produced by multiplying two numbers?

   ....

The number 80 can also be produced by multiplying 2, 10 and 4:
$2 \times 10 = 20$ and $20 \times 4 = 80$ or $10 \times 4 = 40$ and $40 \times 2 = 80$.

We can use brackets to describe what calculation is done first. So instead of writing "$2 \times 10 = 20$ and $20 \times 4 = 80$" we may write $(2 \times 10) \times 4$. Instead of writing "$10 \times 4 = 40$ and 40×2" we may write $2 \times (10 \times 4)$.

2. Show how the number 80 can be produced by multiplying four numbers. Describe how you do it in two ways: without using brackets and by using brackets.

   ....

3. Show three different ways in which the number 30 can be produced by multiplying two numbers.

   ....

4. (a) Can the number 30 be produced by multiplying three whole numbers? ....
   Which three whole numbers? ....
   (b) Can the number 30 be produced by multiplying four whole numbers that do not include the number 1? ....
   If you answered "yes", which four numbers? ....

The number 105 can be produced by multiplying 3, 5 and 7, hence we can write $105 = 3 \times 5 \times 7$. Mathematicians often describe this by saying "105 is the **product** of 3, 5 and 7" or "105 can be **expressed as the product** $3 \times 5 \times 7$".

5. Express each of the following numbers as a product of three numbers.

   (a) 248 .... (b) 375 ....

> The whole numbers that are multiplied to form a number are called **factors** of the number. For example, 6 and 8 are factors of 48 because $6 \times 8 = 48$.

But 6 and 8 are not the only numbers that are factors of 48. 2 is also a factor of 48 because 48 = 2 × 24. And 24 is a factor of 48. The numbers 3 and 16 are also factors of 48 because 48 = 3 × 16.

6. Describe all the different ways in which 48 can be expressed as a product of two factors.

   ....

> The number 36 can be formed by 2 × 2 × 3 × 3.
> Because 2 is used twice, it is called a **repeated factor** of 36. The number 3 is also a repeated factor of 36.

7. (a) Express 48 as a product of three factors. ....
   (b) Express 75 as a product of three factors. ....

8. (a) Can 36 be expressed as a product of three factors? How? ....
   (b) Can 36 be expressed as a product of five factors? How? ....

9. Express each of the following numbers as a product of as many factors as possible, including repeated factors. Do not use 1 as a factor.

   (a) 300 .... (b) 310 ....

   (c) 320 .... (d) 330 ....

   (e) 340 .... (f) 350 ....

PRIME NUMBERS

1. Express each of the following numbers as a product of as many factors as possible, including repeated factors. Do not use 1 as a factor.

   (a) 36 .... (b) 37 ....
   (c) 38 .... (d) 39 ....
   (e) 40 .... (f) 41 ....
   (g) 42 .... (h) 43 ....
   (i) 44 .... (j) 45 ....
   (k) 46 .... (l) 47 ....
   (m) 48 .... (n) 49 ....

2. Which of the numbers in question 1 cannot be expressed as a product of two whole numbers, except as the product *1 × the number itself*?

....

A number that cannot be expressed as a product of two whole numbers, except as the product *1 × the number itself*, is called a **prime number**.

3. (a) Which of the numbers in question 1 are prime? ....

   (b) Which numbers between 20 and 30 are prime? ....

   (c) Are 11 and 17 prime numbers? ....

Eratosthenes, a Greek mathematician who lived a long time ago, designed a method to find the prime numbers. The process is called "the sieve of Eratosthenes".

4. Work on the table on the right.
   Follow the steps to find all the prime numbers up to 100.

   Step 1: Cross out 1.

   Step 2: Circle 2, and then cross out all the multiples of 2.

   Step 3: Circle 3, then cross out all the multiples of 3.

   Step 4: Find the next number that has not been crossed out and cross out all its multiples.

   Continue like this.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

5. (a) What is the smallest number that can be formed as a product of three prime numbers, if the same factor may be repeated? ....

   (b) What is the smallest number that can be formed as a product of three prime numbers, if no repeated factors are allowed? ....

6. Manare did a lot of work, and found out that 840 can be formed as the product of 2, 2, 2, 3, 5 and 7. Check whether Manare is correct.

....

We can say that Manare **found the prime factors** of 840, or Manare **factorised 840 completely**.

We can write:

$2 \times 2 \rightarrow 4 \times 2 \rightarrow 8 \times 3 \rightarrow 24 \times 5 \rightarrow 120 \times 7 = 840$.

7. The prime factors of some numbers are given below. What are the numbers?

   (a) 3, 5, 5 and 11 (b) 3, 3, 5 and 7 (c) 2, 7, 11 and 13

   ....

8. Investigate which of the following statements you agree with. Give reasons for your agreement or disagreement in each case.

   (a) If a number is even, 2 is one of its prime factors. ....

   ....

   (b) If half an even number is also even, 2 is a repeated prime factor. ....

   ....

   ....

   (c) If a number is odd, 3 is one of its prime factors. ....

   ....

   (d) If a number ends in 0 or 5, then 5 is one of its prime factors. ....

   ....

Here is a method to find the prime factors of a number:

If the number is even, divide it by 2. If the answer is even, divide by 2 again. Continue like this as long as it is possible. If the answer is odd, divide by 3, if it is possible. Continue to divide by 3 as long as it is possible. Then switch to 5. Continue like this by each time trying to divide by the next prime number.

9. Find all the prime factors of each of the following numbers. Work in your exercise book or on loose paper, and write only your answers below.

   (a) 588 .... (b) 825 ....

   (c) 729 .... (d) 999 ....

   (e) 538 .... (f) 113 ....

10. Find at least three prime numbers between 800 and 850.

   ....

HIGHEST COMMON FACTOR AND LOWEST COMMON MULTIPLE

1. (a) Factorise 195 and 385 completely.

   ....

   (b) Is 7 a factor of both 195 and 385? ....

   (c) Is 5 a factor of both 195 and 385? ....

When a number is a factor of two or more other numbers, it is called a **common factor** of the other numbers. For example, the number 5 is a common factor of 195 and 385.

The factors of a certain number are 2; 2; 5; 7; 7; 11 and 17. The factors of another number are 2; 3; 3; 7; 7; 11; 13 and 23. The common prime factors of these two numbers are 2; 7; 7 and 11.

The biggest number that is a factor of two or more numbers is called the **highest common factor (HCF)** of the numbers.

2. Find the HCF of the two numbers in each of the following cases.

   (a) $2 \times 2 \times 5 \times 7 \times 7 \times 11 \times 17$ and $2 \times 3 \times 3 \times 7 \times 7 \times 11 \times 13 \times 23$

   ....

   (b) 24 and 40 .... (c) 8 and 12 ....

   (d) 12 and 20 .... (e) 210 and 56 ....

3. Write five different numbers, all different from 35, that have 35 as a highest common factor.

   ....

4. Write the next seven numbers in each pattern:

   A: 12 24 36 48 ....

   B: 15 30 45 60 ....

The numbers in pattern A are called the **multiples** of 12. The numbers in pattern B are called the multiples of 15. The numbers, for example 60 and 120, that occur in both patterns, are called the **common multiples** of 12 and 15. The smallest of these numbers, namely 60, is called the **lowest common multiple** (LCM) of 12 and 15.

5. Continue writing multiples of 18 and 24 below, until you find the LCM:

   18 36 ....

   24 48 ....

6. Find the HCF and LCM of the given numbers in each case below:

   (a) 5 and 7 ....

   (b) 15 and 14 ....

   (c) 20 and 30 ....

   (d) 10 and 100 ....

   (e) 8 and 9 ....

   (f) 25 and 24 ....

   (g) 8 and 12 ....

   (h) 10 and 18 ....

1.4 Properties of operations

ORDER OF OPERATIONS AND THE ASSOCIATIVE PROPERTY

Suppose you want to tell another person to do some calculations. You may do this by writing instructions. For example, you may write the instruction 200 – 130 – 30. This may be called a **numerical expression**.

Suppose you have given the instruction 200 – 130 – 30 to two people, whom we will call Ben and Sara.

This is what Ben does: 200 – 130 = 70 and 70 – 30 = 40.

This is what Sara does: 130 – 30 = 100 and 200 – 100 = 100.

To prevent such different interpretations or understandings of the same numerical expression, mathematicians have made the following agreement, and this is followed all over the world:

> In a numerical expression that involves **addition and subtraction only**, the operations should be performed **from left to right**, **unless otherwise indicated** in some way.

An agreement like this is called a **mathematical convention**.

1. Who followed this convention in the above story, Ben or Sara? ....

2. Follow the above convention and calculate each of the following:

   (a) 8 000 + 6 000 – 3 000 ....

   (b) 8 000 – 3 000 + 6 000 ....

   (c) 8 000 + 3 000 – 6 000 ....

3. Follow the above convention and calculate each of the following:

   (a) R25 000 + R30 000 + R13 000 + R6 000 ....

   (b) R13 000 + R6 000 + R30 000 + R25 000 ....

   (c) R30 000 + R25 000 + R6 000 + R13 000 ....

In question 3, all your answers should be the same. When three or more numbers are added, the order in which you perform the calculations makes no difference. This is called the **associative property of addition**. We also say: **addition is associative**.

4. Investigate whether multiplication is associative. Use the numbers 2, 3, 5 and 10.

   ....

5. What must be added to each of the following numbers to get 100?

   73 56 66 41 34 23 88

   ....

6. Calculate each of the following. Note that you can make the work simple by being smart in deciding which additions to do first.

   (a) 73 + 54 + 27 + 46 + 138 (b) 34 + 88 + 41 + 66 + 59 + 12 + 127

   ....

THE COMMUTATIVE PROPERTY OF ADDITION AND MULTIPLICATION

1. (a) What is the total cost of 20 chairs at R250 each? ....

   (b) What is the total cost of 250 exercise books at R20 each? ....

   (c) R5 000 was paid for 100 towels. What is the price for 1 towel?

   ....

   (d) R100 was paid for 5 000 beads. What is the price for 1 bead?

   ....

2. Which of the following calculations will produce the same answer? Mark those that will produce the same answers with a ✓ and those that won't with a ✗.

   (a) 20×250 and 250×20
   (b) $5\,000 \div 100$ and $100 \div 5\,000$
   (c) $730 + 270$ and $270 + 730$
   (d) $730 - 270$ and $270 - 730$

25 + 75 and 75 + 25 have the same answer. The same is true for any other two numbers. We say: addition is **commutative**; the numbers can be swopped around.

3. Demonstrate each of your answers with two different examples.
   (a) Is subtraction commutative?

   ....

   (b) Is multiplication commutative?

   ....

   (c) Is division commutative?

   ....

MORE CONVENTIONS AND THE DISTRIBUTIVE PROPERTY

1. Do the following:
   (a) Multiply 5 by 3, then add the answer to 20. ....
   (b) Add 5 to 20, then multiply the answer by 5. ....

Mathematicians have agreed that **unless otherwise indicated, multiplication and division should be done before addition and subtraction**. According to this convention, the expression $20 + 5 \times 3$ should be taken to mean "multiply 5 by 3, then add the answer to 20" and not "add 5 to 20, then multiply the answer by 3".

2. Follow the above convention and calculate each of the following:

   (a) $500 + 20 \times 10$ ....
   (b) $500 - 20 \times 10$ ....
   (c) $500 + 20 - 10$ ....
   (d) $500 - 20 + 10$ ....
   (e) $500 + 200 \div 5$ ....
   (f) $500 - 200 \div 5$ ....

   If some of your answers are the same, you have made mistakes.

The above convention creates a problem. How can one describe the calculations in question 1(b) with a numerical expression, without using words?

To solve this problem, mathematicians have agreed to use brackets in numerical expressions. **Brackets are used to specify that the operations within the brackets should be done first.** Hence the numerical expression for 1(b) above is (20 + 5) × 5, and the answer is 125.

If there are **no brackets** in a numerical expression, it **means that multiplication and division should be done first, and addition and subtraction only later.**

If you wish to specify that addition or subtraction should be done first, that part of the expression should be enclosed in brackets.

Examples
The expression 12 + 3 × 5 means "multiply 3 by 5, then add 12". It *does not* mean "add 12 and 3, then multiply by 5".
If you wish to say "add 5 and 12, then multiply by 3", the numerical expression should be 3 × (5 + 12) or (5 + 12) × 3. They mean the same.

3. Keep the various mathematical conventions about numerical expressions in mind when you calculate each of the following:
   (a) 500 + 30 × 10 ....
   (b) (500 + 30) × 10 ....
   (c) 100 × 500 + 30 ....
   (d) 100 × (500 + 30) ....
   (e) 500 – 30 × 10 ....
   (f) (500 – 30) × 10 ....
   (g) 100 × 500 – 30 ....
   (h) 100 × (500 – 30) ....
   (i) (200 + 300) ÷ 20 ....
   (j) 200 ÷ 20 + 300 ÷ 20 ....
   (k) 600 ÷ (20 + 30) ....
   (l) 600 ÷ 20 + 600 ÷ 30 ....

4. Calculate the following:
   (a) 50 × (70 + 30) ....
   (b) 50 × 70 + 50 × 30 ....
   (c) 50 × (70 – 30) ....
   (d) 50 × 70 – 50 × 30 ....

Your answers for 4(a) and 4(b) should be the same.
Your answers for 4(c) and 4(d) should also be the same.

5. Do not do calculations A to I below. Just answer these questions about them. You will check your answers later.
   (a) Will A and B have the same answers? ....
   (b) Will G and H have the same answers? ....
   (c) Will A and D have the same answers? ....
   (d) Will A and G have the same answers? ....
   (e) Will A and F have the same answers? ....
   (f) Will D and E have the same answers? ....

A: $5 \times (200 + 3)$	B: $5 \times 200 + 3$
C: $5 \times 200 + 5 \times 3$	D: $5 + 200 \times 3$
E: $(5 + 200) \times 3$	F: $(200 + 3) \times 5$
G: 5×203	H: $5 \times 100 + 5 \times 103$
I: $5 \times 300 - 5 \times 70$	

6. Now do calculations A to I. Then check the answers you gave in question 5.
   A. ....
   B. ....
   C. ....
   D. ....
   E. ....
   F. ....
   G. ....
   H. ....
   I. ....

7. (a) Choose three different numbers between 3 and 11, and write them down below.
   Your first number: ...... Your second number: ...... Your third number: ......
   (b) Add your first number to your third number. Multiply the answer by your second number.

   ....

(c) Multiply your first number by your second number. Also multiply your third number by your second number. Add the two answers.

....

(d) If you worked correctly, you should get the same answers in (b) and (c). Do you think you will get the same result with numbers between 10 and 100, or any other numbers? ....

The fact that your answers for calculations like those in 7(b) and 7(c) are equal, for any numbers that you may choose, is called the **distributive property of multiplication over addition**.

It may be described as follows:

first number × second number + first number × third number

= first number × (second number + third number).

This can be described by saying that **multiplication distributes over addition**.

8. Check whether the distributive property is true for the following sets of numbers:
   (a) 100, 50 and 10

   ....

   (b) any three numbers of your own choice (you may use a calculator to do this)

   ....

   ....

9. Use the numbers in question 8(a) to investigate whether multiplication also distributes over subtraction.

   ....

   ....

It is quite fortunate that multiplication distributes over addition, because it makes it easier to multiply.

For example, 8 × 238 can be calculated by calculating 8 × 200, 8 × 30 and 8 × 8, and adding the answers: 8 × 238 = 8 × 200 + 8 × 30 + 8 × 8 = 1 600 + 240 + 64 = 1 904.

10. Check whether 8 × 238 is actually 1 904 by calculating 238 + 238 + 238 + 238 + 238 + 238 + 238 + 238, or by using a calculator.

    ....

    ....

1.5 Basic operations

A METHOD OF ADDITION

To add two numbers, the one may be written below the other.

For example, to calculate 378 539 + 46 285 the one number may be written below the other so that the units are below the units, the tens below the tens, and so on.

378 539
46 285

Writing the numbers like this has the advantage that

- the units parts (9 and 5) of the two numbers are now in the same column,
- the tens parts (30 and 80) are in the same column,
- the hundreds parts (500 and 200) are in the same column, and so on.

This makes it possible to work with each kind of part separately.

We only write this:	In your mind you can see this:					
378 539	300 000	70 000	8 000	500	30	9
46 285		40 000	6 000	200	80	5

The numbers in each column can be added to get a new set of numbers:

378 539	300 000	70 000	8 000	500	30	9
46 285		40 000	6 000	200	80	5
14						14
110					110	
700				700		
14 000			14 000			
110 000		110 000				
300 000	300 000					
424 824	It is easy to add the new set of numbers to get the answer.					

Note that you can do the above steps in any order. Instead of starting with the units parts as shown above, you can start with the hundred thousands, or any other parts.

Starting with the units parts has an advantage though: it makes it possible to do more of the work mentally and to write less, as shown below:

378 539	To achieve this, only the units digit 4 of the 14 is written in the first step. The 10 of the 14 is remembered and added to the 30 and 80 of the tens column, to get 120.
46 285	
424 824	

We say the 10 is **carried** from the units column to the tens column. The same is done when the tens parts are added to get 120: only the digit "2" is written (in the tens column, so it means 20), and the 100 is carried to the next step.

1. Calculate each of the following:
   (a) 237 847 + 87 776
   (b) 567 298 + 392 076
   (c) 28 387 + 365 667

A municipal manager is working on the municipal budget for a year. He has to try to keep the total expenditure on new office equipment below R800 000. He still has to budget for new computers that are badly needed, but this is what he has written so far:

74 new office chairs	R 54 020
42 new computer screens	R 100 800
12 new printers	R 141 600
18 new tea trolleys	R 25 740
8 new carpets for senior staff offices	R 144 000
108 small plastic filing cabinets	R 52 380
new table for the boardroom	R 48 000
18 new chairs for the boardroom	R 41 400
	R ....

2. How much has the municipal manager budgeted for printers and computer screens together?

3. How much, in total, has the municipal manager budgeted for chairs and tables?

4. Work out the total cost of all the items the municipal manager has budgeted for.

5. Calculate.
   (a) 23 809 + 2 009 + 23
   (b) 320 293 + 16 923 + 349 + 200 323

METHODS OF SUBTRACTION

There are many ways to subtract one number from another. For example, R835 234 – R687 885 can be calculated by "filling up" from R687 885 to R835 234:

687 885 + 15 → 687 900 + 100 → 688 000 + 12 000 → 700 000 + 135 234 → 835 234

The difference between R687 885 and R835 234 can now be calculated by adding up the numbers that had to be added to 687 885 to get 835 234.

So R835 234 – R687 885 = R147 349.

15
100
12 000
135 234
147 349

Another easy way to subtract is to **round off and compensate**. For example, to calculate R3 224 – R1 885, the R1 885 may be rounded up to R2 000. The calculation can proceed as follows:

- Rounding R1 885 up to R2 000 can be done in two steps: 1 885 + 15 = 1 900, and 1 900 + 100 = 2 000. In total, 115 was added.
- 115 can now be added to 3 224 too: 3 224 + 115 = 3 339.

Instead of calculating R3 224 – R1 885, which is a bit difficult, R3 339 – R2 000 may be calculated. This is easy: R3 339 – R2 000 = R1 339.

This means that R3 224 – R1 885 = R1 339, because R3 224 – R1 885 = (R3 224 + R115) – (R1 885 + R115).

To do question 1, you may use any one of the above two methods, or any other method you may know and prefer. Do not use a calculator, because the purpose of this work is for you to come to understand how subtraction may be done. What you will learn here, will later help you to understand **algebra**.

1. Calculate each of the following:

(a) 6 234 – 2 992

(b) 76 214 – 34 867

(c) 134 372 – 45 828

(d) 623 341 – 236 768

2. Check each of your answers in question 1 by doing addition, or by doing subtraction with a different method than the method you have already used.

....

....

....

Another method of subtraction is to think of the numbers in **expanded notation**. For example, to calculate R835 234 – R687 885, which was already done in a different way on the previous page, we could work like this:

We may write this:	In your mind you can see this:					
835 234	800 000	30 000	5 000	200	30	4
687 885	600 000	80 000	7 000	800	80	5

Unfortunately, it is not possible to subtract in the columns now. However, the parts of the bigger number can be rearranged to make the subtraction in each column possible:

835 234	700 000	120 000	14 000	1100	120	14
687 885	600 000	80 000	7 000	800	80	5
	100 000	40 000	7 000	300	40	9

The answer is now clearly visible; it is 147 349.

The rearrangement, also called "borrowing", was done like this:
10 was taken from the 30 in the tens column, and added to the 4 in the units column.
100 was taken from the 200 in the hundreds column, and added to the 20 that remained in the tens column. 1 000 was taken from the 5 000 in the thousands column, and added to the 100 that remained in the hundreds column.

3. Describe the other rearrangements that were made in the above work.

....

....

....

....

It is not practical to write the expanded notation and the rearrangements each time you do a subtraction. However, with some practice you can learn to do it all in your mind without writing it down. Some people make small marks above the digits of the bigger number, or even change the digits, to keep track of the rearrangements they make in their minds.

835 234
687 885
147 349

4. Calculate the difference between the two car prices in each case.
   (a) R73 463 and R88 798
   (b) R63 378 and R96 889

5. In each case, first estimate the answer to the nearest 100 000, then calculate.
   (a) 238 769 – 141 453
   (b) 856 333 – 439 878

6. In each case, first estimate the answer to the nearest 10 000, then calculate.
   (a) 88 023 – 45 664
   (b) 342 029 – 176 553

7. Look again at the municipal budget on page 35. How much money does the municipal manager have left to buy new computers?

8. Calculate.
   (a) 670 034 – 299 999
   (b) 670 034 – 300 000
   (c) 376 539 – 175 998
   (d) 376 541 – 176 000

A METHOD OF MULTIPLICATION

6 × R3 258 can be calculated in parts, as shown below.

6 × R3 000 = R18 000

6 × R200 = R1 200

6 × R50 = R300

6 × R8 = R48

$$\begin{array}{r} 3\ 2\ 5\ 8 \\ \times\ \ 6 \\ \hline 4\ 8 \\ 3\ 0\ 0 \\ 1\ 2\ 0\ 0 \\ 1\ 8\ 0\ 0\ 0 \\ \hline 1\ 9\ 5\ 4\ 8 \end{array}$$

The four partial products can now be added to get the answer, which is R19 548. It is convenient to write the work in vertical columns for units, tens, hundreds and so on, as shown on the right above.

In fact, if you are willing to do some hard thinking you can produce the answer with even less writing. You can achieve this by working from right to left to calculate the partial products, and by "carrying" parts of the partial answers to the next column, as you can do when working from right to left in columns. It works like this:

$$\begin{array}{r} 3\ 2\ 5\ 8 \\ \times\ \ 6 \\ \hline 1\ 9\ 5\ 4\ 8 \end{array}$$

When 6 × 8 = 48 is calculated, only the "8" is written down, in the units column. The "4" that represents 40 is not written. It is kept "on hold" in your mind.

When 6 × 50 = 300 is calculated, the 40 from the previous step is added to 300 to get 340. Again, only the "4" that represents 40 is written. The 300 is kept on hold or "carried" to add to the answer of the next step. The work continues like this.

1. Calculate each of the following. Do not use a calculator.

   (a) 8 × 786

   (b) 9 × 3 453

   (c) 60 × 786

   (d) 60 × 7 860

2. You may use a calculator to check your answers for question 1. Repeat the work if your answers are not correct, so that you can learn where you make mistakes. Then put your calculator away again.

3. Use your answers for questions 1(a) and (c) to find out how much 68 × 786 is.

To calculate 36 × 378, the work can be broken up in two parts, namely 30 × 378 and 6 × 378.

4. Calculate 36 × 378.

A complete write-up of calculating 76 × 2 348 in columns is shown on the right.

			2	3	4	8
				×	7	6
A					4	8
B				2	4	0
C			1	8	0	0
D		1	2	0	0	0
E				5	6	0
F			2	8	0	0
G		2	1	0	0	0
H	1	4	0	0	0	0
	1	7	8	4	4	8

5. (a) Explain how the 240 in row B was obtained.

   (b) Explain how the 560 in row E was obtained.

   (c) Explain how the 21 000 in row G was obtained.

A short write-up of calculating 76 × 2 348 in columns is shown on the right.

You may try to do the calculations in question 6 in this way. If you find it difficult, you may first write some of them up completely, and then try again to write less when you multiply.

		2	3	4	8
			×	7	6
	1	4	0	8	8
1	6	4	3	6	0
1	7	8	4	4	8

6. Calculate each of the following.

   (a) 53 × 738

   (b) 73 × 3 457

7. Calculate.

(a) 64 × 3 478

....

....

....

....

....

(b) 78 × 1 298

....

....

....

....

....

(c) 37 × 3 428

....

....

....

....

....

(d) 78 × 7 285

....

....

....

....

....

8. Use a calculator to check your answers for question 7. Redo the questions that you had wrong, so that you can learn to work correctly.

9. Use your correct answers for question 7 to give the answers to the following, without doing any calculations:

(a) 101 244 ÷ 1 298 ....

(b) 568 230 ÷ 7 285 ....

10. Calculate, without using a calculator.

(a) 3 659 × 38

....

....

....

....

....

(b) 27 × 23 487

....

....

....

....

....

(c) 486 × 278

....

....

....

....

....

....

(d) 2 135 × 232

....

....

....

....

....

....

A PROCESS CALLED LONG DIVISION

You may use a calculator to do questions 1 to 6.

1. You want to buy some live chickens at R37 each and you have R920 available. How many live chickens can you buy in total?

   ....

2. R880 is to be shared equally among 34 learners? How many full rands can each learner get?

   ....

3. You want to buy live chickens at R47 each. You have R1 280 available. How many live chickens can you buy?

   ....

4. 42 equal bags of rice weigh a total of 7 560 g. How much does one bag weigh?

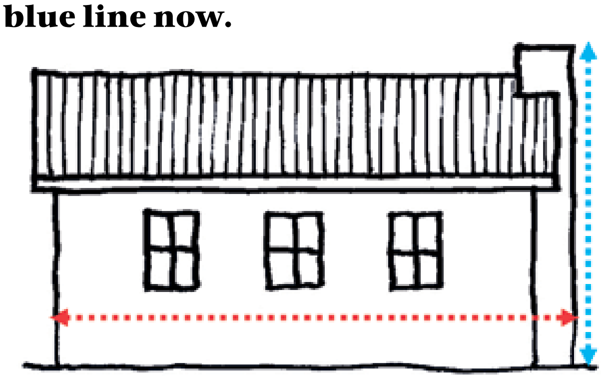
   ....

5. The number 26 was multiplied by a secret number and the answer was 2 184. What was the secret number?

   ....

This is an accurate sketch of the back of a house. The red line on the sketch is 70 mm long and it shows the width of the house. The blue line on the sketch indicates the height of the chimney. **Do not measure the blue line now.**

The width of the actual house is 5 600 mm, and the height of the chimney is 3 360 mm.



6. (a) How many times is the house bigger than the sketch? Describe what you can do to find this out.

   ....

   ....

   (b) Calculate how long the blue line on the sketch should be.

   ....

   (c) Now measure the blue line to check your answer for (b). ....

Division is used for different purposes:

> In question 1 you knew that the amount is split into equal parts. You had to **find out how many parts there are** (how many chickens). This is called **grouping**.
>
> In question 2 you knew that the amount was split into 34 equal parts. You needed to **find out how big each part is** (how much money each learner will get). This is called **sharing**.

7. (a) What does question 3 require, sharing or grouping? ....

   (b) What does question 4 require, sharing or grouping? ....

In question 6 division was done for a different purpose than sharing or grouping.

Put your calculator away now. It is very important to be able to solve division problems by using your own mind. The activities that follow will help you to do this better than before. While you work on these activities, you will often have to **estimate** the product of two numbers. If you can estimate products well, division becomes easier to do. Hence, to start, do question 8, which will provide you with opportunities to practise your product estimation skills.

8. (a) What do you think is closest to 4 080:
   10 × 74 or 30 × 74 or 50 × 74 or 70 × 74 or 90 × 74? ....

   (b) Calculate some of the products to check your answer.

   ....

   (c) What do you think is closest to 9 238: 30 × 38 or 50 × 38 or 100 × 38 or 150 × 38 or 200 × 38 or 250 × 38 or 300 × 38? ....

   (d) Calculate some of the products to check your answer.

   ....

   (e) What do you think is closest to 9 746: 10 × 287 or 20 × 287 or 30 × 287 or 40 × 287 or 50 × 287 or 60 × 287 or 70 × 287? ....

   (f) Calculate some of the products to check your answer.

   ....

   (g) By what multiple of 10 should you multiply 27 to get as close to 6 487 as possible?

   ....

9. A principal wants to buy T-shirts for the 115 Grade 7 learners in the school. The T-shirts cost R67 each, and an amount of R8 500 is available. Do you think there is enough money to buy T-shirts for all the learners? Explain your answer.

....

10. (a) How much will 100 of the T-shirts cost? ....

(b) How much money will be left if 100 T-shirts are bought? ....

(c) How much money will be left if 20 more T-shirts are bought? ....

The principal wants to work out exactly how many T-shirts, at R67 each, she can buy with R8 500. Her thinking and writing are described below.

Step 1

What she writes:	What she thinks:
$67\overline{)\,8\,500}$	*I want to find out how many chunks of 67 there are in 8 500.*

Step 2

What she writes:	What she thinks:
100	*I think there are at least 100 chunks of 67 in 8 500.*
$67\overline{)\,8\,500}$	
$\underline{6\,700}$	*100 × 67 = 6 700. I need to know how much is left over.*
1 800	*I want to find out how many chunks of 67 there are in 1 800.*

Step 3 (She has to rub out the one "0" of the 100 on top, to make space.)

What she writes:	What she thinks:
120	*I think there are at least 20 chunks of 67 in 1 800.*
$67\overline{)\,8\,500}$	
$\underline{6\,700}$	
1 800	
$\underline{1\,340}$	*20 × 67 = 1 340. I need to know how much is left over.*
460	*I want to find out how many chunks of 67 there are in 460.*

Step 4 (She rubs out another "0".)

What she writes:	What she thinks:
125	*I think there are at least 5 chunks of 67 in 460.*
$67\overline{)\,8\,500}$	
$\underline{6\,700}$	
1 800	
$\underline{1\,340}$	
460	
$\underline{335}$	*5 × 67 = 335. I need to know how much is left over.*
125	*I want to find out how many chunks of 67 there are in 125.*

Step 5 (She rubs out the "5".)

What she writes:	What she thinks:
126	*I think there is only one more chunk of 67 in 125.*
67 ⟌ 8 500	
6 700	
1 800	
1 340	
460	
335	
125	
67	*I wonder how much money will be left over.*
58	*So, we can buy 126 T-shirts and R58 will remain.*

Do not use a calculator in the questions that follow. The purpose of this work is for you to develop a good understanding of how division can be done. Check all your answers by doing multiplication.

11. (a) Selina bought 85 chickens, all at the same price. She paid R3 995 in total. What did each of the chickens cost? Your first step can be to work out how much Selina would have paid if she paid R10 per chicken, but you can start with a bigger step if you wish.

(b) Anton has R4 850. He wants to buy some young goats. The goats cost R78 each. How many goats can he buy?

12. Calculate the following without using a calculator:

(a) 7 234 ÷ 48

(b) 3 267 ÷ 24

(c) 9 500 ÷ 364

(d) 8 347 ÷ 24

13. (a) A chocolate factory made 9 325 chocolates of a very special kind one day. The chocolates were packed in small decorated boxes, with 24 chocolates per box. How many boxes were filled?

(b) A farmer sells eggs packed in cartons to the local supermarkets. There are 36 eggs in one carton. One month, the farmer sold 72 468 eggs to the supermarkets. How many cartons is this?

1.6 Problem solving

RATE AND RATIO

You may use a calculator for doing the work in this section.

1. The people in a village get their water from a nearby dam. On a certain day the dam contains 688 000 litres of water. The village people use about 85 000 litres of water each day. For how many days will the water in the dam last, if no rains fall?

   ....

Instead of saying "85 000 litres each day" or "8 cm each hour", people often say "**at a rate of** 85 000 litres **per day**" or "**at a rate of** 8 cm **per hour**".

2. During a period of very heavy rain, the water level in a certain river increases at a rate of 8 cm each hour. If it continues like this, by how much will the water level increase in 24 hours?

   ....

3. A woman is driving from Johannesburg to Durban. Her distance from Durban decreases at a rate of about 95 km per hour. How far does she travel, approximately, in 4 hours?

   ....

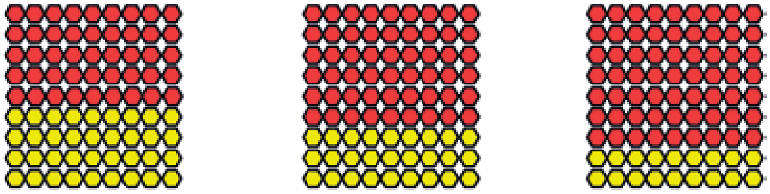
4. The number of unemployed people in a certain province increases at a rate of approximately 35 000 people per year. If there were 860 000 unemployed people in the year 2000, how many unemployed people will there be, approximately, in the year 2020?

   ....

5. In pattern A below, there are 5 red beads for every 4 yellow beads.
   Describe patterns B and C in the same way.

   ....

   ....



Pattern A *Pattern B* *Pattern C*

In a certain food factory, two machines are used to produce tins of baked beans. Machine A produces at a rate of 800 tins per hour, and machine B produces at a rate of 2 400 tins per hour.

6. (a) Complete the following table, to show how many tins of beans will be produced at the two machines, in different periods of time.

Number of hours	1	2	3	5	8
Number of tins at machine A	800	1 600	2 400	4 000	
Number of tins at machine B	2 400	4 800			

(b) How much faster is machine B than machine A? ....

(c) How many tins will be produced at machine B in the time that it takes machine A to produce 30 tins? ....

(d) How many tins will be produced at machine B in the time that it takes machine A to produce 200 tins? ....

(e) How many tins will be produced at machine B in the time that it takes machine A to produce 1 tin? ....

The patterns in question 5 can be described like this:

In pattern A, the **ratio** of yellow beads to red beads is 4 to 5. This is written as 4 : 5.

In pattern B, the ratio between yellow beads and red beads is 3 : 6, and in pattern C the ratio is 2 : 7. In question 6, machine A produces 1 tin for every 3 tins that machine B produces. This can be described by saying that the ratio between the production speeds of machines A and B is 1 : 3.

7. Two huge trucks are travelling very slowly on a highway. Truck A covers 20 km per hour, and truck B covers 30 km per hour. Both trucks keep these speeds all the time.

(a) What distance will truck B cover in the same time that truck A covers 10 km? ....

(b) In the table below, the distances that truck A covers in certain periods of time are given. Complete the table, to show the distances covered by truck B, in the same periods of time.

Distance covered by truck A	10 km	18 km	50 km	100 km	30 km
Distance covered by truck B					

(c) What distance will truck B cover in the same time that truck A covers 1 km? ....

(d) What is the ratio between the speed at which truck A travels and the speed at which truck B travels? ....

8. R240 will be divided between David and Sally in the ratio 3 : 5. This means Sally gets R5 for every R3 David gets. How much will David and Sally each get in total?

....

....

9. How much will each person get, if R14 400 is shared between two people in each of the following ways?
   (a) In the ratio 1 : 3

   ....

   ....

   (b) In the ratio 5 : 7

   ....

   ....

FINANCIAL MATHEMATICS

A man borrows R12 000 from a bank for one year. He has to pay 15% interest to the bank. This means that, apart from paying the R12 000 back to the bank after a year, he has to pay 15 hundredths of R12 000 for the privilege of using the money that actually belongs to the bank.

One hundredth of R12 000 can be calculated by dividing R12 000 by 100. This amount can then be multiplied by 15 to get 15 hundredths of R12 000.

15% is read as **15 per cent**, and it is just a different way to say **15 hundredths**.

The money paid for using another person's house is called **rent**. The money paid for using another person's money is called **interest**.

Do not use a calculator when you do the following questions.

1. Calculate 12 000 ÷ 100, then multiply the answer by 15.

   ....

   ....

2. Calculate:
   (a) 12% of R8 000 (b) 18% of R24 000

   ....

   ....

3. In each case below, calculate how much interest must be paid.
   (a) An amount of R6 000 is borrowed for 1 year at 9% interest.

   ....

   (b) An amount of R21 000 is borrowed for 3 years at 11% interest per year.

   ....

   (c) An amount of R45 000 is borrowed for 10 years at 12% interest per year.

   ....

A car dealer buys a car for R60 000 and sells it for R75 000. The difference of R15 000 is called the **profit**. In this case, the profit is a quarter of R60 000, which is the same as 25 hundredths or 25%. This can be described by saying "the car dealer made a profit of 25%".

4. Calculate the amount of profit in each of the following cases. The information is about a car dealer who buys and sells used vehicles.
   (a) A car is bought for R40 000 and sold for R52 000.

   ....

   (b) A small truck is bought for R100 000 and sold at a profit of 28%.

   ....

   (c) A bakkie is bought for R120 000 and sold at a profit of 30%.

   ....

A shop owner bought a stove for R2 000 and sold it for R1 600. The shop owner did not make a profit, he sold the stove at a **loss** of R400.

5. (a) How much is 1 hundredth of R2 000? ....
   (b) How many hundredths of R2 000 is R400? ....
   (c) How much is 20% of R2 000? ....

Notice that by doing question 5(b) you have worked out at what percentage loss the shop owner sold the stove.

6. The shop owner also sold a fridge that normally sells for R4 000 at a **discount** of 20%. This means the customer paid 20% less than the normal price. Calculate the discount in rands and the amount that the customer paid for the fridge.

   ....

   ....

Chapter 2
Exponents

In this chapter, you will learn about a very short way to describe calculations like this:

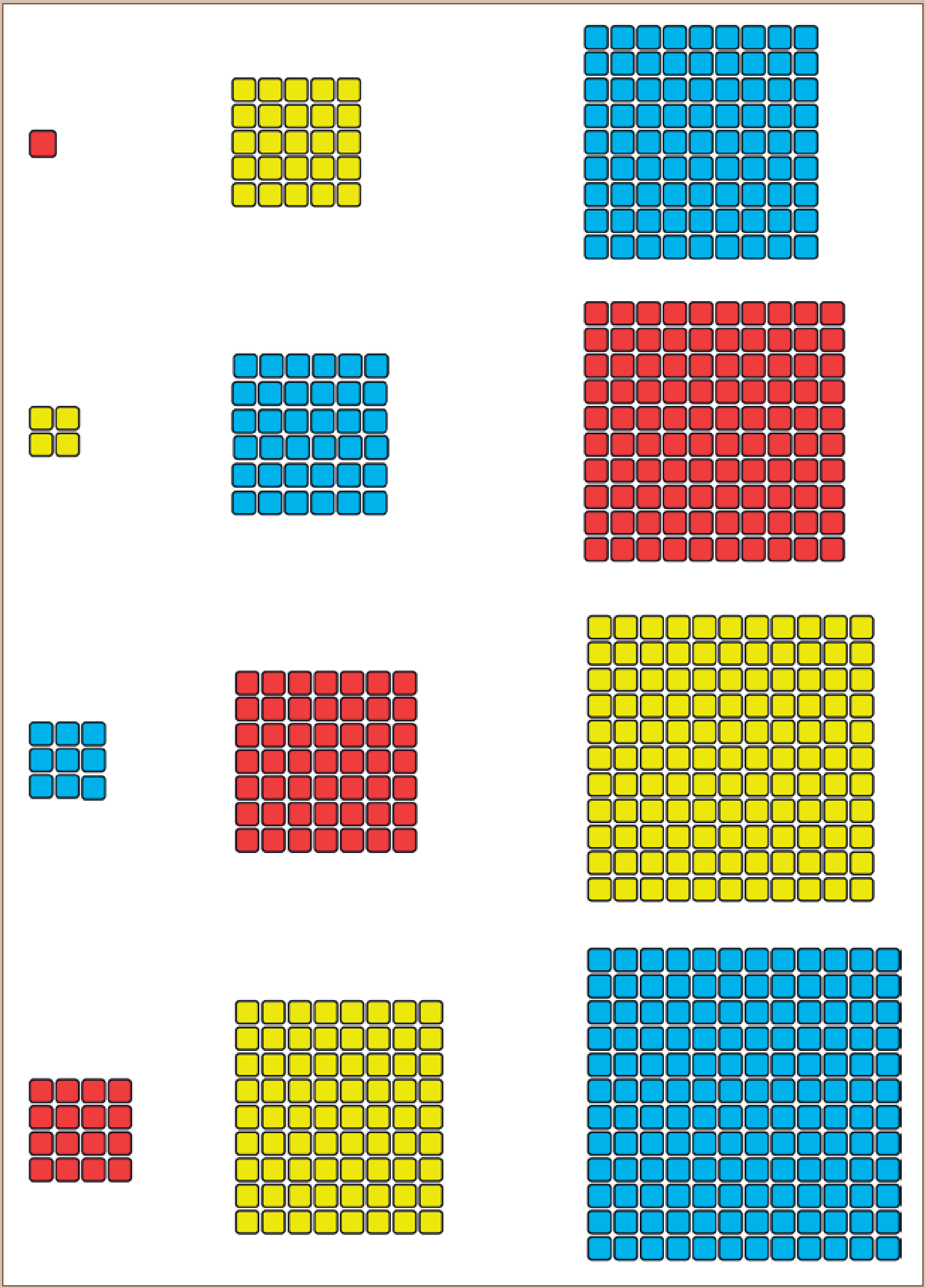
$$3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$$

You already know a short way to describe calculations like this:

$$3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3$$

5	$5 + 5$	$5 + 5 + 5$	$5 + 5 + 5 + 5$	$5 + 5 + 5 + 5 + 5$	$5 + 5 + 5 + 5 + 5 + 5$
1×5	2×5	3×5	4×5	5×5	6×5

5	5×5	$5 \times 5 \times 5$	$5 \times 5 \times 5 \times 5$	$5 \times 5 \times 5 \times 5 \times 5$	$5 \times 5 \times 5 \times 5 \times 5 \times 5$
5^1	5^2	5^3	5^4	5^5	5^6



2 Exponents

2.1 Quick squares and cubes

AGAIN AND AGAIN

1. How much is each of the following?

   2×2 3×3 4×4 5×5 6×6 7×7

   ....

   8×8 9×9 10×10 11×11 12×12 1×1

   ....

Instead of saying “ten times ten”, we may say “ten squared” and we may write 10^2.

2. Complete the tables.

2×2			12×12		8×8
2^2	5^2			4^2	
2 squared		10 squared			
	25	100			64

		1×1	9×9		
	7^2		9^2		
11 squared				3 squared	
121					36

3. 8 squared is 64, and 9 squared is 81.

   (a) What number squared is 25? .....
   (b) What number squared is 100? .....
   (c) What number squared is 64? .....
   (d) What number squared is 36? .....

4. Calculate:

   (a) $10^2 + 5^2 + 2^2$

   ....

   (b) $5 \times 10^2 + 7 \times 10 + 3$

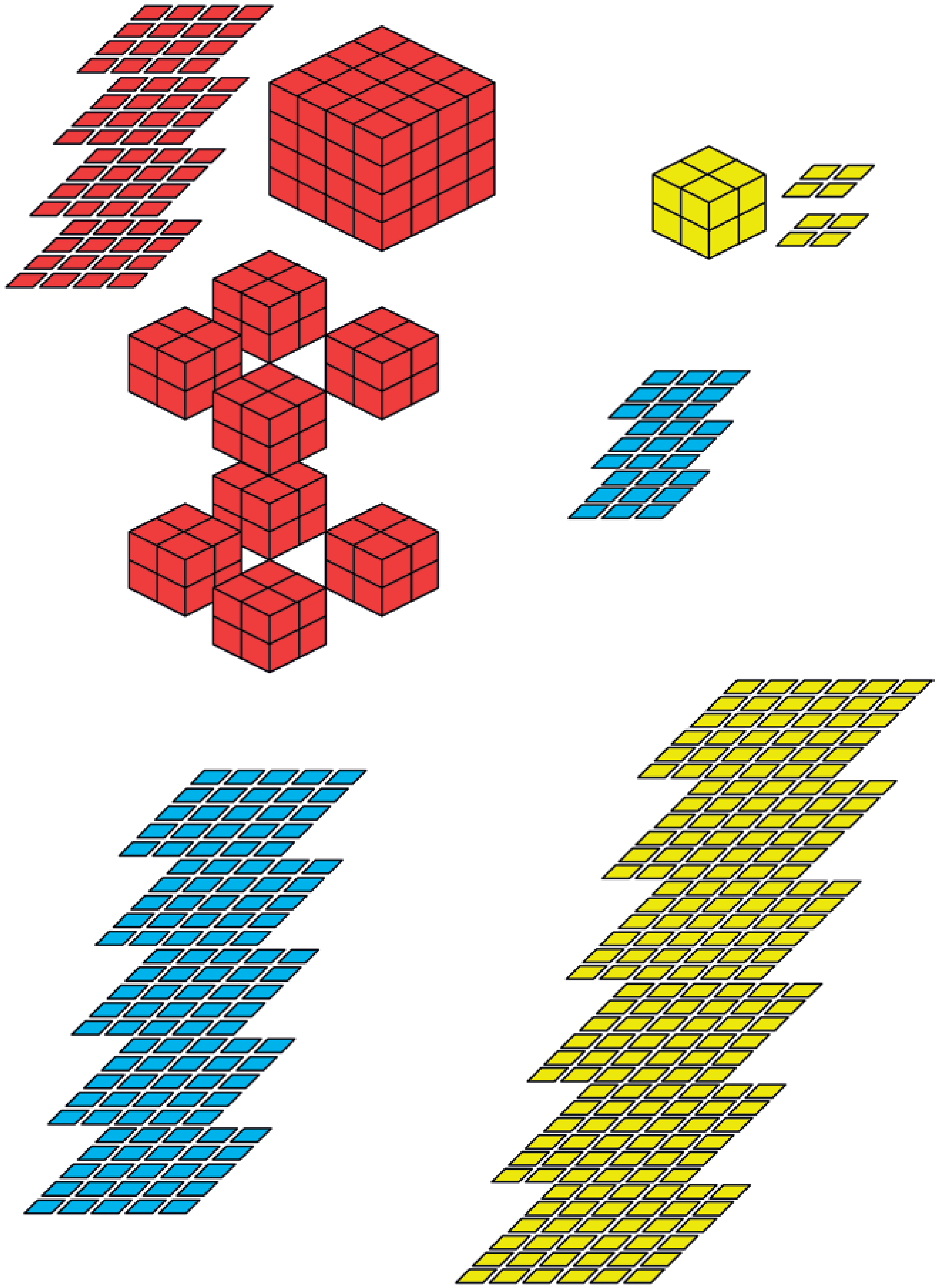
   ....

   (c) $7 \times 10^2 + 3 \times 10 + 6$

   ....

   (d) $2 \times 10^2 + 9 \times 10 + 6$

   ....



5. How much is each of the following?

$2 \times 2 \times 2$ $3 \times 3 \times 3$ $4 \times 4 \times 4$ $5 \times 5 \times 5$ $6 \times 6 \times 6$

....

$7 \times 7 \times 7$ $8 \times 8 \times 8$ $9 \times 9 \times 9$ $10 \times 10 \times 10$

....

$11 \times 11 \times 11$ $12 \times 12 \times 12$ $13 \times 13 \times 13$ $1 \times 1 \times 1$

....

Instead of saying "10 times 10 times 10", we may say "10 cubed" and we may write 10^3.

6. Complete the tables.

$4 \times 4 \times 4$	$7 \times 7 \times 7$				
4^3		11^3			
4 cubed			2 cubed		
64				216	1 000

$8 \times 8 \times 8$					
			9^3		
	12 cubed			3 cubed	
		1			125

7. 5 cubed is 125, and 9 cubed is 729.
   (a) What number cubed is 27? .....
   (b) What number cubed is 1 000? .....
   (c) What number cubed is 8? .....
   (d) What number cubed is 1? .....
   (e) What number cubed is 216? .....
   (f) What number cubed is 343? .....

8. Calculate:
   (a) $3 \times 10^3 + 7 \times 10^2 + 5 \times 10 + 6$

   ....

   (b) $7 \times 10^3 + 7 \times 10^2 + 7 \times 10 + 7$

   ....

   (c) $8 \times 10^3 + 1 \times 10^2 + 4 \times 10 + 2$

   ....

   (d) $4 \times 10^3 + 3 \times 10^2 + 4 \times 10 + 9$

   ....

   (e) 10×10^2

   ....

   (f) $10^2 \times 10^2$

   ....

9. Can you think of two numbers, so that the square of the one number is equal to the cube of the other number?

   ....

10. Can you think of two numbers, so that when you add their squares, you get the square of another number?

   ....

2.2 The exponential notation

REPEATED MULTIPLICATION WITH THE SAME NUMBER

1. Express each number below as a product of prime factors.
   Example: $250 = 2 \times 5 \times 5 \times 5$

(a)	35	....	(b)	70	....
(c)	140	....	(d)	280	....
(e)	81	....	(f)	625	....

5 is a **repeated factor** of 250. It is repeated 3 times.

2. Which numbers in question 1 have repeated factors? In each case, state what number is repeated as a factor and how many times it is repeated.

   ....

   ....

A number that can be expressed as a product of one repeated factor is called a **power** of that number.

Examples:

32 is a power of 2, because $32 = 2 \times 2 \times 2 \times 2 \times 2$

100 000 is a power of 10, because $10 \times 10 \times 10 \times 10 \times 10 = 100\,000$

3. Express each number as a power of 2, 3, 5 or 10.

(a)	125	(b)	64
....		....	
(c)	100	(d)	1 000
....		....	

4. Calculate each of the following. You can use each answer to get the next answer.

(a) $2 \times 2 \times 2 \times 2$ ....

(b) $2 \times 2 \times 2 \times 2 \times 2$ ....

(c) $2 \times 2 \times 2 \times 2 \times 2 \times 2$ ....

(d) $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$ ....

(e) $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

....

(f) $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

....

(g) $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

....

(h) $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

....

(i) $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

....

(j) $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

....

Because the factor 2 is repeated 5 times, 32 is called the **fifth power of 2**, or **2 to the power 5**.

Similarly, 125 is the third power of 5.

125 can also be called "5 to the power 3" or "5 cubed".

5. The seventh power of 2 is shown in question 4(d).
What power of 2 is shown in each of the following parts of question 4?

(a) 4(j) ....

(b) 4(i) ....

(c) 4(h) ....

(d) 4(f) ....

6. What power of what number is shown in each case below?

(a) $15 \times 15 \times 15 \times 15 \times 15 \times 15 \times 15 \times 15$

....

(b) $12 \times 12 \times 12 \times 12 \times 12 \times 12 \times 12 \times 12 \times 12 \times 12 \times 12 \times 12$

....

Instead of writing "5 to the power 6" we may write 5^6.
This is called the **exponential notation**.
5^6 means $5 \times 5 \times 5 \times 5 \times 5 \times 5$.
5×6 means $6 + 6 + 6 + 6 + 6$.

7. Write each of the numbers in question 3 in exponential notation.

(a) .... (b) ....

(c) .... (d) ....

8. Write each of the numbers in question 4 in exponential notation.

(a) .... (b) ....

(c) .... (d) ....

(e) .... (f) ....

(g) .... (h) ....

(i) .... (j) ....

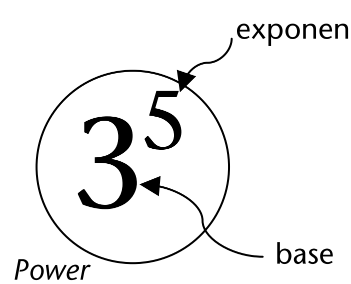
9. In each case write the number in exponential notation.

(a) The fifth power of 5 .... (b) The sixth power of 5 ....

(c) The third power of 4 .... (d) 6 to the power 4 ....

(e) 4 to the power 6 .... (f) 5 to the power 15 ....

3^5 means $3 \times 3 \times 3 \times 3 \times 3$.
The repeating factor in a power is called the **base**.
The number of repetitions is called the **exponent** or **index**.
3^1 means 3. The base is 3 but there is no repetition.
Any number raised to the power 1 equals the number itself.



10. In each case below some information about a number is given. Each number can be expressed as a power. What is the number in each case?

(a) The base is 5 and the index is 3. ....

(b) The base is 10 and the exponent is 4. ....

(c) The base is 20 and the exponent is 3. ....

11. Calculate each of the following:

(a) $5 \times 5 \times 5$ .... (b) $5 \times 5 \times 5 \times 5 \times 5$ ....

(c) $5 + 5 + 5$ .... (d) $5 + 5 + 5 + 5 + 5$ ....

(e) 5×3 .... (f) 5^3 ....

POWERS OF DIFFERENT NUMBERS

1. Complete this table of powers of 2.
   (You have already calculated these powers on page 57.)

Exponent	1	2	3	4	5	6	7	8	9
Power of 2	2	4	8	16					

Exponent	10	11	12	13	14
Power of 2					

2. (a) Calculate each of the following:
   $2^2 - 2^1$ $2^3 - 2^2$ $2^4 - 2^3$ $2^5 - 2^4$ $2^6 - 2^5$ $2^7 - 2^6$ $2^8 - 2^7$

   ....

   (b) Describe what you notice about the differences between consecutive powers of 2.

   ....

Numbers that follow on each other in a pattern are called **consecutive numbers**.

3. Suppose you calculate the differences between consecutive powers of 3. Do you think these differences will be the consecutive powers of 3 again?

   ....

4. Complete this table of powers of 3.

Exponent	1	2	3	4	5	6	7	8	9
Power of 3	3	9							

Exponent	10	11	12	13	14
Power of 3					

5. (a) Calculate each of the following:
   $3^2 - 3^1$ $3^3 - 3^2$ $3^4 - 3^3$ $3^5 - 3^4$ $3^6 - 3^5$ $3^7 - 3^6$ $3^8 - 3^7$

   ....

   (b) How do these numbers differ from what you expected when you answered question 3?

   ....

   (c) Divide each of your answers in 5(a) by 2.

   ....

   (d) If you observe anything interesting, describe it.

   ....

6. In questions 1 to 5 you have investigated the differences between consecutive powers of 2 and 3. You have observed certain interesting things about these differences. You will now investigate, in the same way, the differences between consecutive powers of 4.

   (a) Before you investigate, think a bit. What do you expect to find?

   ....

   ....

   (b) Do your investigation, and write a short report on what you find.

Exponent								
Power of 4								

....

....

....

....

....

....

....

....

7. Do what you did in question 6, but now for powers of 10.

Exponent							
Power of 10							

....

....

....

....

....

....

....

....

2.3 Squares and cubes

The number 9 is called the **square** of 3 because $3 \times 3 = 9$. The number 3, called the base, is multiplied by itself. $\mathbf{3^2}$ is read as **three squared** or **three to the power 2**.

The number 27 is called the **cube** of 3 because $3 \times 3 \times 3 = 27$. The base, the number 3, is multiplied by itself and again by itself. $\mathbf{3^3}$ is read as **three cubed** or **three to the power 3**.

CALCULATING SQUARES AND CUBES

Squaring the number 2 means that we must multiply 2 by itself. It means we have to calculate 2×2, which has a value of 4, and we write $2 \times 2 = 4$.

1. In (a) to (f) below, the numbers in set B are found by squaring each number in set A. Write down the numbers that belong to set B in each case.

	Set A	Set B
(a)	{1; 2; 3; 4; 5; 6; 7; 8}	
(b)	{1; 3; 5; 7; 9; 11; 13}	
(c)	{10; 20; 30; 40; 50}	
(d)	{2; 4; 6; 8; 10; 12; 14}	
(e)	{5; 10; 15; 20; 25}	
(f)	{15; 12; 9; 6; 3}	

Cubing the number 2 means that we must multiply 2 by itself, and again. It means we have to calculate $2 \times 2 \times 2$, which has a value of 8, and we write $2 \times 2 \times 2 = 8$.

2. (a) Cube 1. Also cube 2 and 3.

....

....

(b) Cube 5. Also cube 10 and 4.

....

....

3. In (a) and (b) below, the numbers in set B are found by cubing each number in set A. Write down the numbers that belong to set B in each case.

   (a) **Set A:** {1; 2; 3; 4; 5; 6; 7; 8}

   Set B: ....

   (b) **Set A:** {10; 20; 30; 40; 50}

   Set B: ....

4. (a) Write down the squares of the first 15 natural numbers.

   ....

   ....

   (b) What do you observe about the last digit of each square number?

   ....

   (c) Give an example of a number that ends in one of the digits you have written above that is not a square.

   ....

The number 64 can be written both as a square and a cube.

$64 = 8^2$ and $64 = 4^3$

The number 17 is neither a square nor a cube.

5. Are the following numbers squares, cubes, both or neither? Just write *square, cube, both* or *neither* where appropriate. Compare your answers with the answers of two classmates.

(a) 64	(b) 1	(c) 121
....	....	....
(d) 1 000	(e) 512	(f) 400
....	....	....
(g) 65	(h) 216	(i) 169
....	....	....

2.4 The square root and the cube root

The inverse to finding the square of a number is to find its **square root**.

The question, "What is the square root of 25?" is the same as the question, "What number, when squared, equals 25?"

The answer to the question is 5 because $5 \times 5 = 25$.

DETERMINING WHAT NUMBER WAS SQUARED

1. What number, when squared, equals 9? Explain.

2. What is the square root of 49? Explain.

3. What number, when squared, equals 81? Explain.

4. What number, when squared, equals 225? Explain.

5. What is the square root of 121? Explain.

6. What number must be squared to get 169? Explain.

7. Complete the diagrams below.

$$14 \longrightarrow \boxed{\text{squared}} \longrightarrow ?$$

$$? \longleftarrow \boxed{\text{square root}} \longleftarrow 196$$

The inverse operation to finding the cube of a number is to find its **cube root**.

The question, "What number, when cubed, equals 125?" is the same as the question, "What is the cube root of 125?"

The answer to the question above is 5 because $125 = 5 \times 5 \times 5$.

DETERMINING WHAT NUMBER WAS CUBED

1. What number, when cubed, equals 27? Explain.

2. What is the cube root of 343? Explain.

3. What number, when cubed, equals 8? Explain.

4. What is the cube root of 1 000? Explain.

5. What number, when cubed, equals 512? Explain.

6. What number produces the same answer when it is squared and when it is cubed?

7. Complete the diagrams below.

$$? \longrightarrow \boxed{\text{cubed}} \longrightarrow 1\ 000$$

$$10 \longleftarrow \boxed{\text{cube root}} \longleftarrow ?$$

CALCULATING SQUARE ROOTS AND CUBE ROOTS

1. Complete the table. The first one has been done for you.

	Number	Cube root	Check your answer
(a)	8	2	$2 \times 2 \times 2 = 8$
(b)	27		
(c)	64		
(d)	125		
(e)	216		
(f)	1 331		
(g)	1 000		
(h)	512		
(i)	8 000		

2. Complete the table. The first one has been done for you.

	Number	Square root	Check your answer
(a)	9	3	$3 \times 3 = 9$
(b)	1 600		
(c)	144		
(d)	196		
(e)	625		
(f)	900		
(g)	16		
(h)	400		
(i)	121		

The symbol $\sqrt{25}$ can be used to indicate the square root of 25. So we can write $\sqrt{25} = 5$.

The symbol $\sqrt[3]{125}$ can be used to indicate the cube root of 125. So we can write $\sqrt[3]{125} = 5$.

3. What mathematical symbol can be used to indicate each of the following?

(a) The square root of 169 .... (b) The cube root of 343 ....

(c) The square root of 2 500 .... (d) The cube root of 729 ....

(e) The cube of 25 .... (f) The square of 25 ....

By agreement amongst mathematicians, the symbol $\sqrt{\ }$ means the square root of the number that is written inside the symbol. So we normally write $\sqrt{4}$ instead of $\sqrt[2]{4}$.

For the cube root, however, the number 3 outside of the root sign $\sqrt[3]{\ }$ must be written in order to distinguish the cube root from the square root.

4. Find the values of each of the following. The first one has been done for you. Check your answers.

		Value	Check your answer
(a)	$\sqrt{64}$	8	$8 \times 8 = 64$
(b)	$\sqrt{49}$		
(c)	$\sqrt{36}$		
(d)	$\sqrt{784}$		
(e)	$\sqrt{2\ 025}$		
(f)	$\sqrt{324}$		

5. Find the values of each of the following. The first one has been done for you. Check your answers.

		Value	Check your answer
(a)	$\sqrt[3]{8}$	2	$2 \times 2 \times 2 = 8$
(b)	$\sqrt[3]{64}$		
(c)	$\sqrt[3]{512}$		
(d)	$\sqrt[3]{1}$		
(e)	$\sqrt[3]{216}$		
(f)	$\sqrt[3]{125}$		

2.5 Comparing numbers in exponential form

BIGGER, SMALLER OR EQUAL?

1. Which is bigger?
   (a) 2^5 or 5^2

   ....

   (b) 3^4 or 4^3

   ....

   (c) 2^3 or 6^1

   ....

We can use mathematical symbols to indicate whether a number is bigger, smaller or has the same value as another number.

We use the symbol > to indicate that the number on the left-hand side of the symbol is bigger than the one on the right-hand side. **The number 5 is bigger than 3 and we express this in mathematical language as 5 > 3.**

The symbol < is used to indicate that the number on the left-hand side of the symbol is smaller than the number on the right-hand side. **The number 3 is smaller than 5 and we express this mathematically as 3 < 5.**

When numbers have the same value we use the equal sign, =. **The numbers 2^3 and 8 have the same value and we write this as $2^3 = 8$.**

2. Use the symbols =, < or > to make the following true. Check your answers.

   (a) $\sqrt[3]{64}$ ☐ $\sqrt{16}$ (b) 3^3 ☐ 4^2

   ....

   (c) 6 ☐ $\sqrt{36}$ (d) $\sqrt[3]{125}$ ☐ $\sqrt{100}$

   ....

   (e) 3^3 ☐ $\sqrt[3]{216}$ (f) 2^4 ☐ 3^4

   ....

(g) 2^3 ☐ 3^2 (h) $\sqrt[3]{1}$ ☐ $\sqrt{1}$

....

(i) 9 ☐ 3^3 (j) 100 ☐ 15^2

....

3. Which is bigger, 1^{100} or 100^1? Explain.

....

4. What is the biggest number you can make with the symbols 4 and 2?

....

5. Two whole numbers that follow on each other, like 4 and 5, are called consecutive numbers. Is the difference between the squares of two consecutive whole numbers always an odd number?

....

BE SMART WHEN DOING CALCULATIONS

Our knowledge of squares can help us to do some calculations much quicker. Suppose you want to calculate 11 × 12.

11^2 has a value of 121. We know that 11 × 11 = 121.

11 × 12 means that there are 12 elevens in total.

So 11 × 12 = 11 × 11 + 11
= 121 + 11
= 132

Suppose you want to calculate 11 × 17.

11 × 17 = 17 elevens in total = 11 elevens + 6 elevens

Well, we know that 11 × 11 = 121

So 11 × 17 = 11 × 11 + 6 × 11
= 121 + 66
= 187

Now do the following calculations in your exercise book, using your knowledge of square numbers.

1. 11 × 19 2. 13 × 16 3. 15 × 18 4. 12 × 18

ARRANGING NUMBERS IN ASCENDING AND DESCENDING ORDER

The numbers 1, 4, 9, 16, 25, ... are arranged from the smallest to the biggest number. We say that the numbers 1, 4, 9, 16, 25, ... are arranged in **ascending order**.

The numbers 25, 16, 9, 4, 1, ... are arranged from the biggest to the smallest number. We say that the numbers 25, 16, 9, 4, 1, ... are arranged in **descending order**.

1. Arrange the following numbers in ascending order:

   (a) $\sqrt[3]{64}$; 3^2; $\sqrt{64}$; $\sqrt{36}$

   ....

   (b) $\sqrt{225}$; $\sqrt[3]{729}$; $\sqrt[3]{1\,000}$; 2^2

   ....

   (c) $\sqrt[3]{1}$; 0; 100; 10^3

   ....

   (d) 1^2; 2^3; 4^2; 5^2

   ....

2. Arrange the following numbers in descending order:

   (a) $\sqrt[3]{216}$; $\sqrt[3]{10^3}$; 2^5; 20

   ....

   (b) 10^3; $\sqrt[3]{20^3}$; $\sqrt{144}$; 12^2

   ....

   (c) $\sqrt{121}$; $\sqrt[3]{125}$; 11^2; 5^3

   ....

   (d) 1^5; 2^4; 7^2; 6^3; 5^3

   ....

2.6 Calculations

THE ORDER OF OPERATIONS

When a numerical expression includes multiple operations, for example both multiplication and addition, what you do first makes a difference.

If there are no brackets in a numerical expression, it means that **multiplication and division must be done first, and addition and subtraction only later**. For example, the expression $12 + 3 \times 5$ means "multiply 3 by 5; then add 12". It does *not* mean "add 12 and 3; then multiply by 5".

If you wish to specify that addition **should be done first**, that part of the expression should be **put in brackets**. For example, if you wish to say "add 5 and 12; then multiply by 3", the numerical expression should be $3 \times (5 + 12)$ or $(5 + 12) \times 3$.

Here is another example: The expression $10 - 6 \div 3$ means "divide 6 by 3; then subtract the answer from 10". It does *not* mean "subtract 6 from 10; then divide by 3". If you wish to specify that subtraction should be done first, that part of the expression should be put in brackets. The numerical expression $(10 - 6) \div 3$ means "subtract 6 from 10; then divide the answer by 3".

It is important to know the **correct order** in which operations in a numerical expression should be done.

WRITING NUMERICAL EXPRESSIONS IN WORDS

Write each of the following numerical expressions in words:

1. $5 \times 2^2 + 3$ ....

   ....

2. $5^2 \times (2 + 3)^2$ ....

   ....

3. $\sqrt{36 + 64} + 3^3$ ....

   ....

4. $\sqrt{16} + \sqrt{9}$ ....

....

5. $10^3 - 9^3$ ....

....

6. $(18 \div \sqrt{9})^2$ ....

....

7. $\dfrac{26-\sqrt{4}}{6}$ ....

....

CALCULATIONS WITH EXPONENTS

Do these calculations without using a calculator.

1. Calculate:

(a) $2^4 + 1^4$

....

....

(b) $(2 + 1)^4$

....

....

(c) $2^3 + 3^3 + 4^3$

....

....

....

(d) $2^3 + 5^3 \times 3$

....

....

....

(e) $12^2 \div 2^3$

....

....

....

(f) $\dfrac{12 + 2 \times 3^2}{4^2 - 1^3}$

....

....

....

2. Do the calculations below and then say which expression has the same value as 2^5.

(a) $2^3 + 2^2$

....

....

....

(b) $2^3 \times 2^2$

....

....

....

3. Do the calculations below and then say which expression has the same value as 5^4.

(a) $5^3 + 5^1$ (b) $5^3 \times 5^1$

....

....

4. Which of the expressions below has the same value as 8^4?

(a) $2^4 \times 4^4$ (b) $8^3 \times 8$

....

....

5. Calculate the following:

(a) $4^2 + 3^2$ (b) $12^2 + 5^2$

....

....

6. (a) Continue this list to find the values of the "powers of 2" from 2^1 to 2^{12}:

$2^1 = 2$; $2^2 = 4$; $2^3 = 8$; $2^4 = 16$; ....

....

(b) Do you notice a pattern in the last digit of the numbers? Write down the pattern in your own words.

....

....

(c) Use the pattern to predict the *last digit* of the following values. (You should not need to actually calculate the values in full.)

(i) 2^{20} (ii) $2^{1\,002}$

....

CALCULATIONS INVOLVING SQUARE ROOTS AND CUBE ROOTS

1. Calculate each of the following without using a calculator:

(a) $\sqrt{64} + \sqrt{36}$ .... (b) $\sqrt{9 + 16}$ ....

(c) $\sqrt{25}$ .... (d) $\sqrt{100}$ ....

(e) $\sqrt{64 + 36}$ .... (f) $\sqrt{9} + \sqrt{16}$ ....

2. Say whether each of the following is true or false. Explain your answer. (*Note*: ≠ in question (d) means "is not equal to")

(a) $\sqrt{64+36} = \sqrt{64} + \sqrt{36}$

. . . .

. . . .

(b) $\sqrt{16} + \sqrt{9} = \sqrt{16+9}$

. . . .

. . . .

(c) $\sqrt{100} = \sqrt{64} + \sqrt{36}$

. . . .

. . . .

(d) $\sqrt{25} \neq \sqrt{9} + \sqrt{16}$

. . . .

. . . .

(e) $\sqrt{9\times 9} = 9$

. . . .

. . . .

(f) $\sqrt[3]{2\times 2\times 2} = 2$

. . . .

. . . .

(g) $\sqrt{169} - \sqrt{25} = 8$

. . . .

. . . .

(h) $\sqrt{169-25} = 12$

. . . .

. . . .

3. Calculate each of the following without using a calculator:

(a) $2 + \sqrt[3]{8} + (3+2)^2$

. . . .

. . . .

(b) $2 + \sqrt[3]{8} + 3^2 + 2^2$

. . . .

. . . .

(c) $2 + \sqrt[3]{8} + 2^5 - 2^3$

. . . .

. . . .

(d) $\dfrac{5 + 4 \times (\sqrt{169} - 2^3)}{5}$

. . . .

. . . .

(e) $(15 - \sqrt{25})^3$

. . . .

. . . .

(f) $\dfrac{28 - 24 \div \sqrt{4}}{(\sqrt[3]{27}+1)^2}$

. . . .

. . . .

WORKSHEET

1. Write in expanded form:

   6^6 ....

2. Write in exponential form:

   14 to the power 9 ....

3. Rewrite the numbers from the smallest to the biggest: 3^4; 2^5; 4^3; 10

   ....

4. Say whether each of the following is true or false. Explain your answer.

   (a) $\sqrt{64+36} = \sqrt{64} + \sqrt{36}$

   ....

   ....

   (b) $\sqrt{25} + \sqrt{9} = \sqrt{59+5}$

   ....

   ....

5. Calculate:

   (a) $3^3 \times 2^2$

   ....

   (b) $\sqrt{144} + \sqrt{81}$

   ....

   (c) $11^2 + 5^2 - \sqrt{144}$

   ....

   (d) $(14-12)^4 \div \sqrt[3]{8}$

   ....

   (e) $9^2 - 4^2 \times 3$

   ....

   (f) $7 + \sqrt[3]{125} + 1^5 - 2^3$

   ....

   (g) $(\sqrt[3]{27} + \sqrt{64})^2$

   ....

   ....

   (h) $(\sqrt{16+9} \div 5^1) \times 93$

   ....

   ....

   (i) $\dfrac{9^2 + 12^2 + 5^3 + 650}{\sqrt[3]{125} \times 10^2}$

   ....

   ....

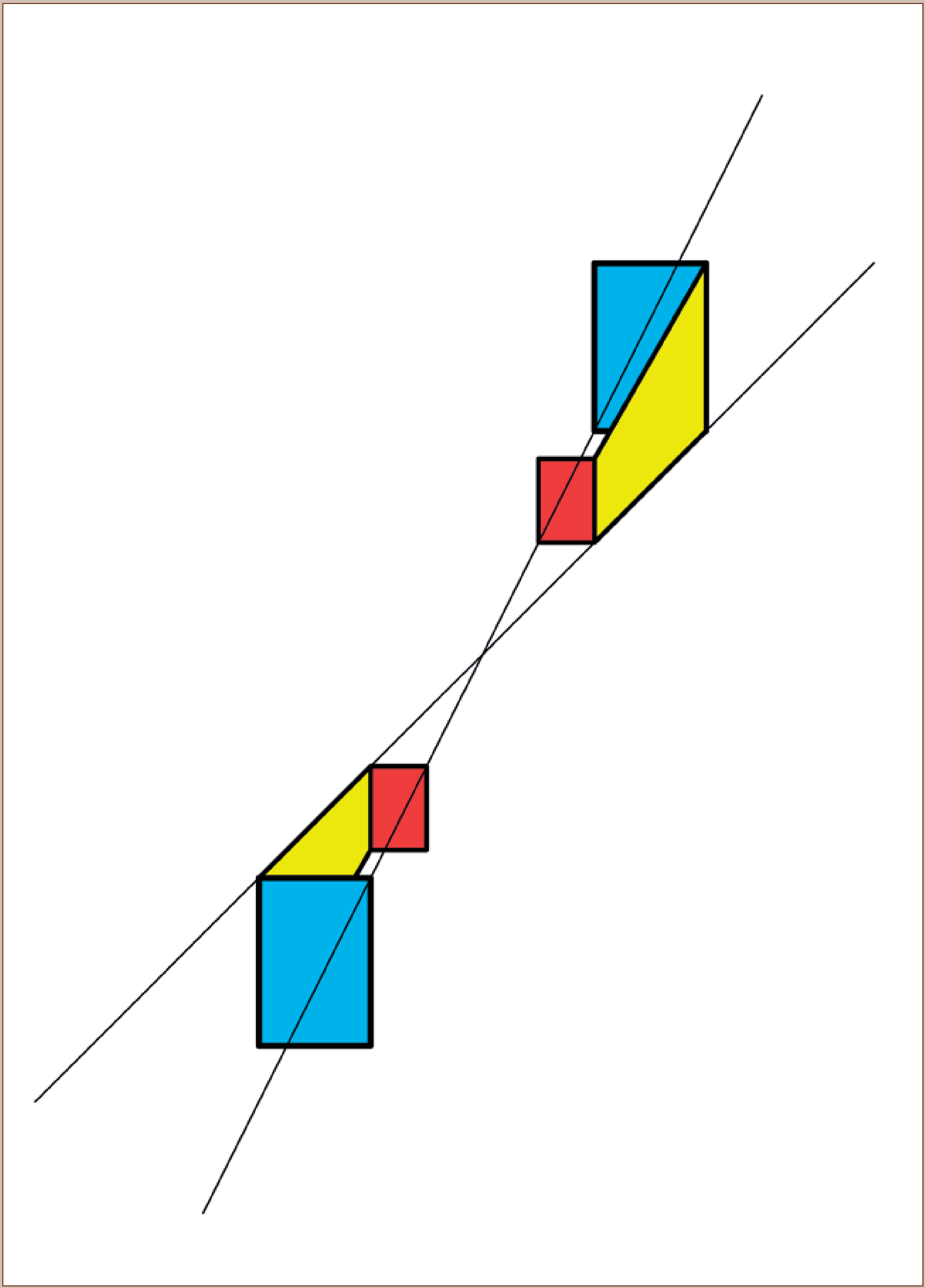
   (j) $\dfrac{6^3 - (\sqrt{169})^2 + \sqrt[3]{8}}{7^2 \times 1^9}$

   ....

   ....

CHAPTER 3
Geometry of straight lines

You probably know exactly what is meant by a "line". In this chapter, you will learn about line segments and rays and how they differ from lines. You will also learn more about parallel and perpendicular lines and how we indicate them on a diagram.

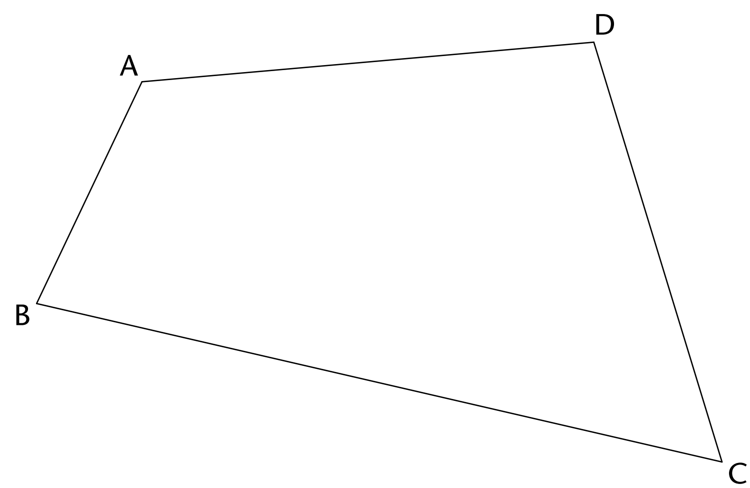


3 Geometry of straight lines

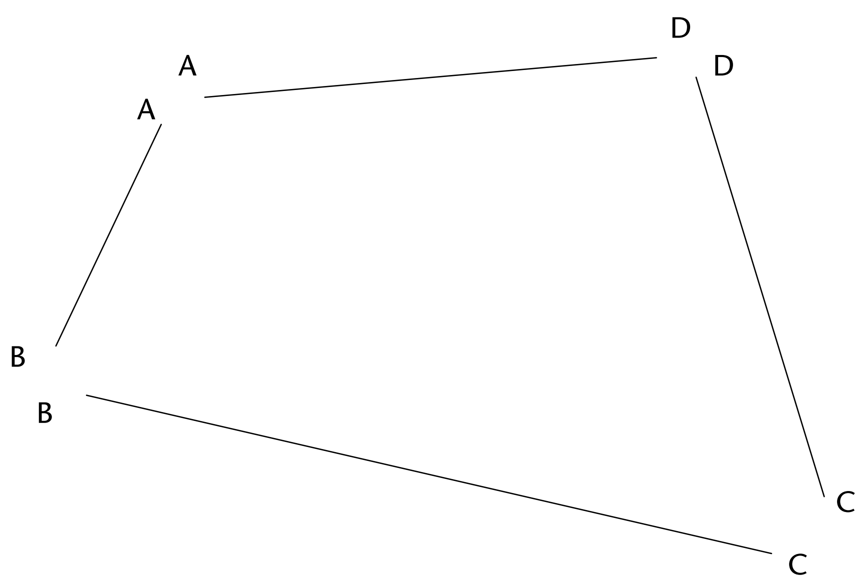
3.1 Line segments, lines and rays

LINE SEGMENTS

1. Measure each side of this quadrilateral. Write the measurements at each side.



Each side of a quadrilateral is a **line segment**.

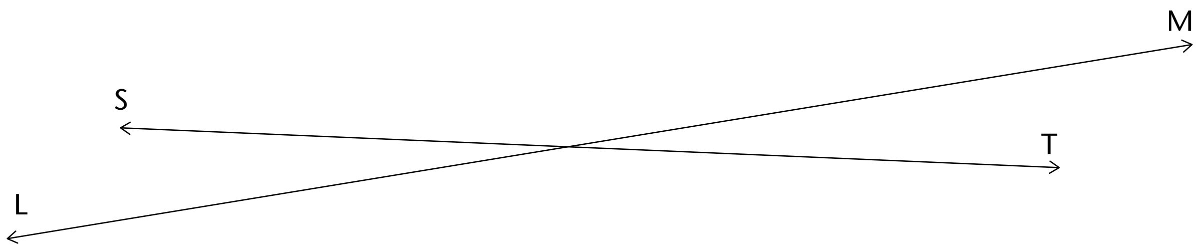


A **line segment** has a definite starting point and a definite endpoint. We can draw and measure line segments.

2. Draw a line segment that is 12 cm long.

LINES AND RAYS

We can think of lines that have no ends, although we cannot draw them completely. We draw line segments to represent lines. When we draw a line segment to represent a line, we may put arrows at both ends to show that it goes on indefinitely on both sides.



The word **line** is used to indicate a line that goes on in both directions. We can only see and draw part of a line. A line cannot be measured.

1. Draw line AB.

2. Did you draw the whole of line AB? Explain.

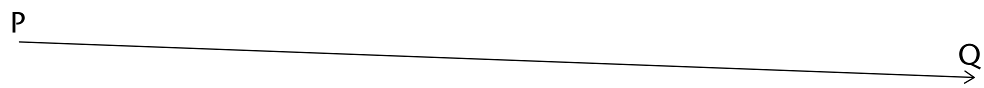
   ....

   ....

We can also think of a line that has a definite starting point but goes on indefinitely at the other end. This is called a half-line or a **ray**.

We can draw the starting point and part of a ray, using an arrow to indicate that it goes on at the one end.

Ray PQ goes on towards the right:

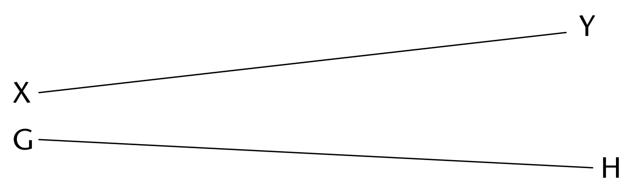


Ray DC goes on towards the left:

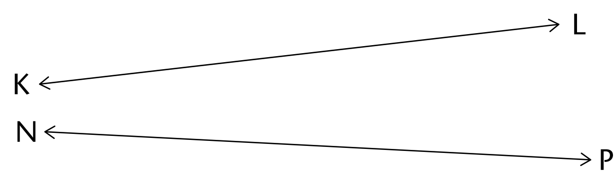
3. Draw ray EF.

4. Did you draw the whole of ray EF? Explain.

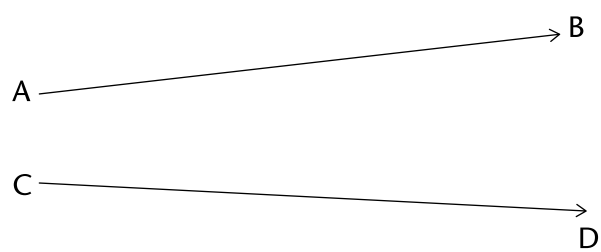
5. Do line segments XY and GH meet anywhere?



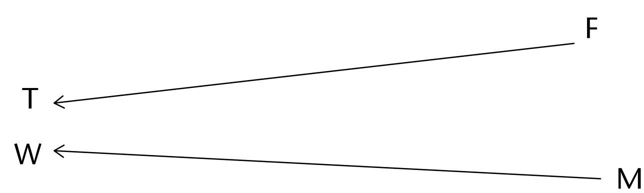
6. Do lines KL and NP meet anywhere?



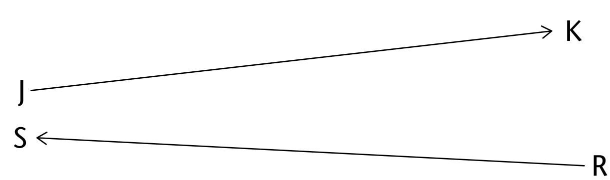
7. Do rays AB and CD meet anywhere?



8. Do rays FT and MW meet anywhere?



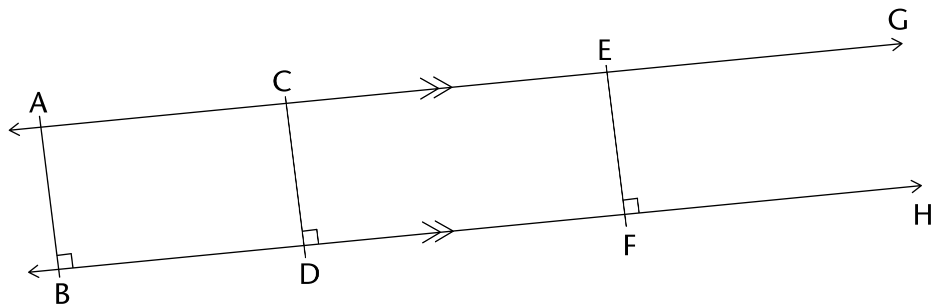
9. Do rays JK and RS meet anywhere?



3.2 Parallel and perpendicular lines

PARALLEL LINES

Two lines that are a constant distance apart are called **parallel lines**. Lines AG and BH below are parallel. The symbol || is used to indicate parallel lines. We write: AG || BH.



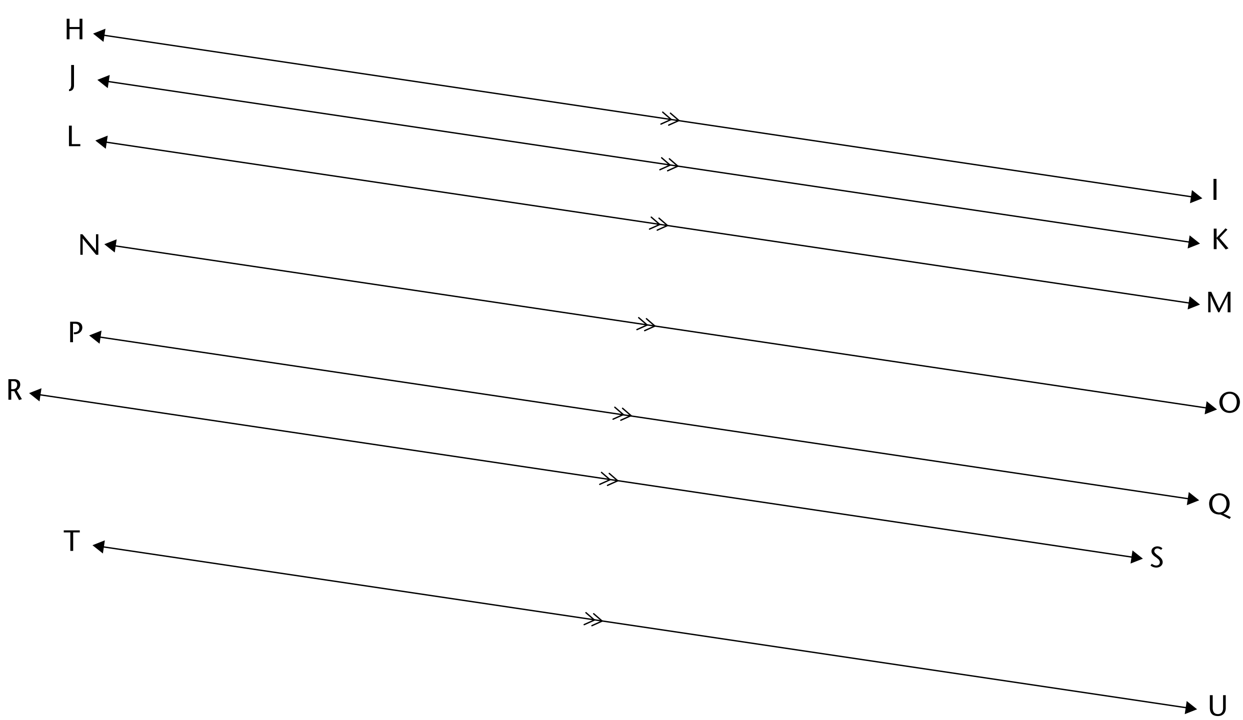
1. Measure the distance between the two lines:

   (a) at A and B ....

   (b) at C and D ....

   (c) at E and F ....

Here are some more parallel lines:



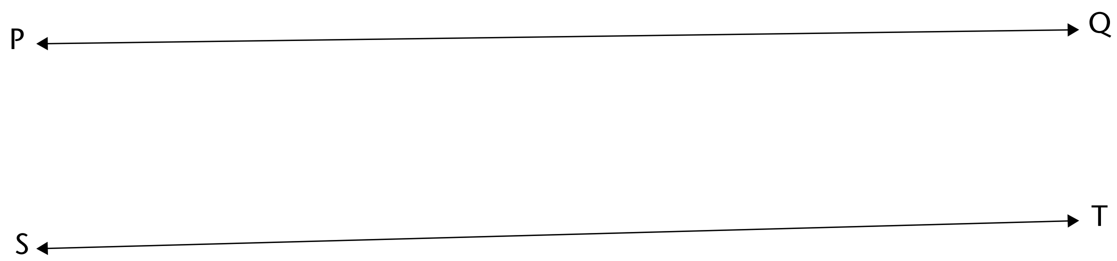
2. Draw two parallel lines.

3. Draw three lines that are parallel to each other.

4. Will parallel lines meet somewhere?

....

5. Do you think lines PQ and ST are parallel? How can you check?



....

....

....

6. (a) Draw two lines that are almost parallel, but not quite.

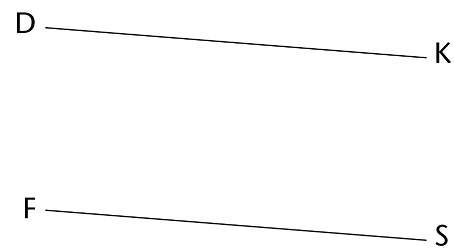
(b) Describe what you did to make sure that your two lines are not parallel.

....

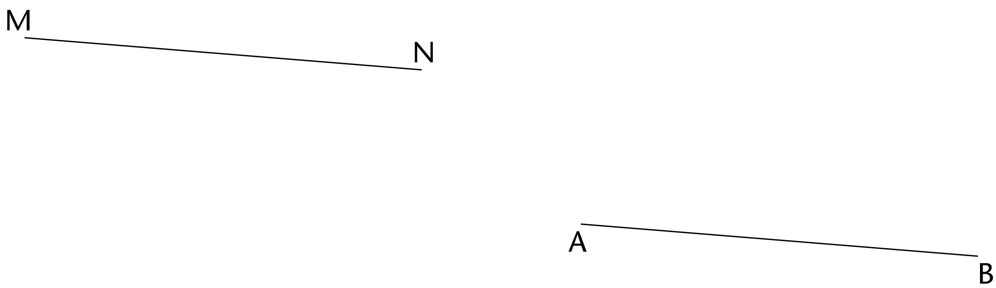
....

7. Can two line segments be parallel? ....

8. Are line segments DK and FS parallel?

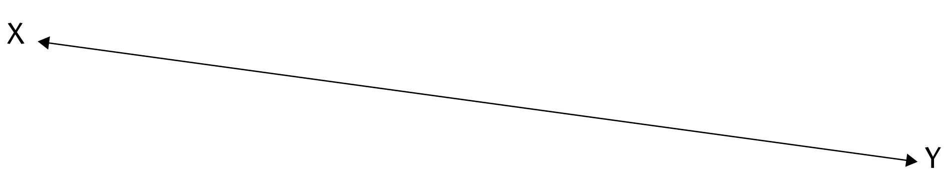


9. Are line segments MN and AB parallel?



10. What can you do so that you will be better able to check whether the above two line segments are parallel or not?

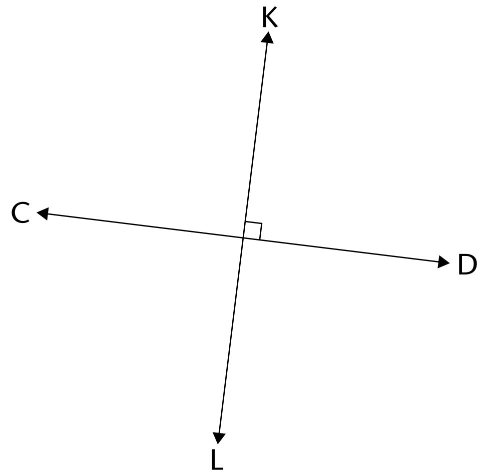
11. Can a line be parallel on its own?



12. Draw a line that is parallel to line XY above.

PERPENDICULAR LINES

Lines CD and KL below are perpendicular to each other. The symbol ⊥ is used to indicate perpendicular lines. We write: CD ⊥ KL.



1. How many angles are formed at the point where the above two lines meet?

   ....

Two lines that form right angles are **perpendicular** to each other.

2. Draw two rays that have the same starting point.

3. Draw two rays that are perpendicular to each other and have the same starting point.

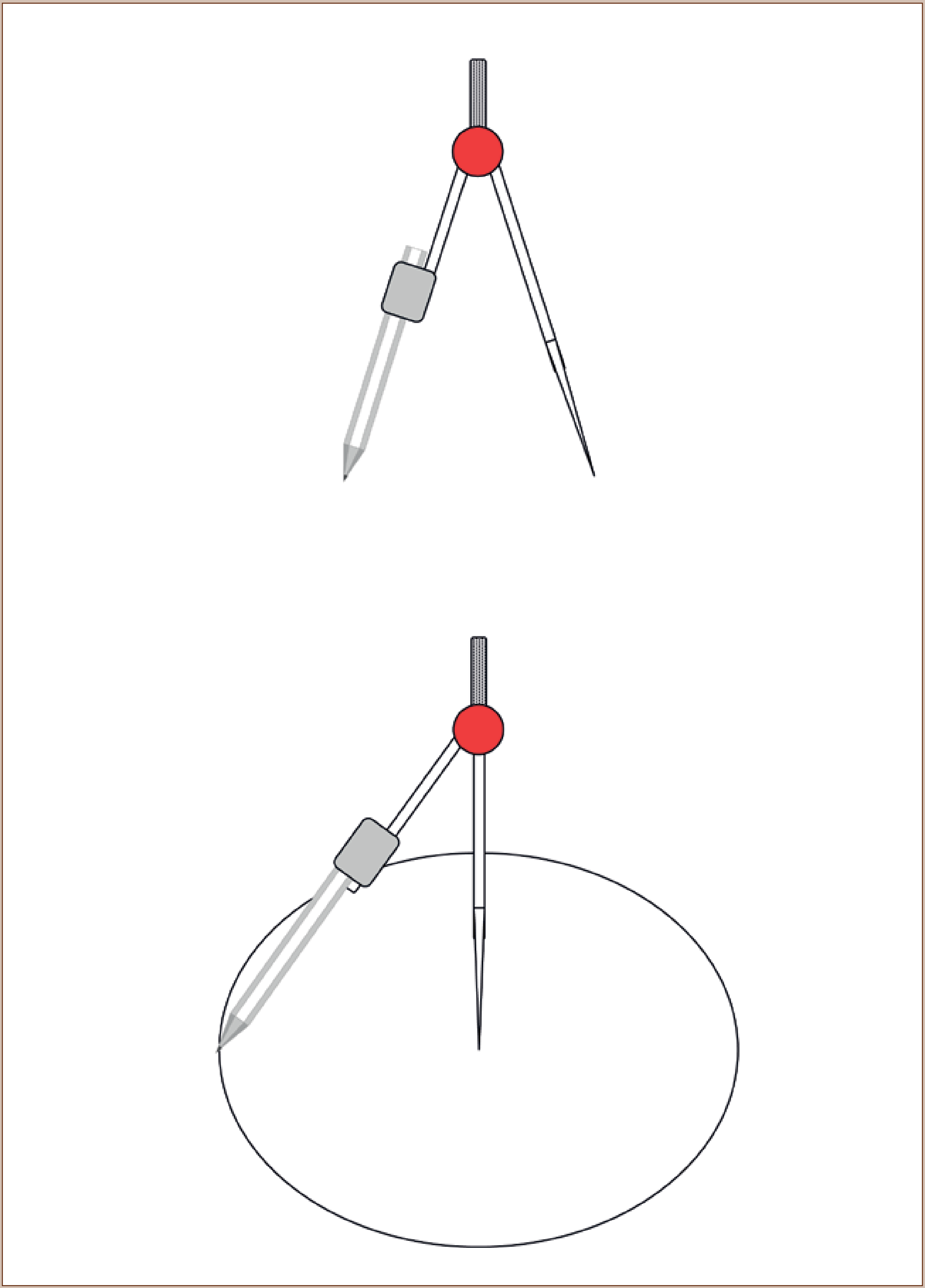
4. Draw two rays that meet, but not at their starting points.

5. Draw two rays that meet but not at their starting points, and that are perpendicular to each other.

6. Can you draw two rays that have the same starting point, and are parallel to each other?

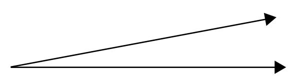
Chapter 4
Construction of geometric figures

In this chapter, you will learn to draw geometric figures accurately. You will also explore the properties that different figures have.

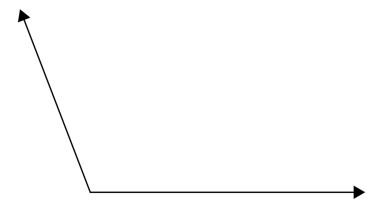


4 Construction of geometric figures

4.1 Angles revision



small angle

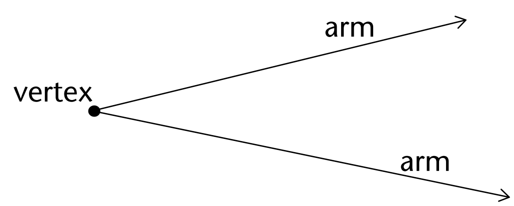


big angle

When two lines point in different directions, we say they are **at an angle** to each other. If the directions are almost the same, we say the **angle** between them is small. If the directions are very different, we say the angle between them is big.

Words we use to describe angles:

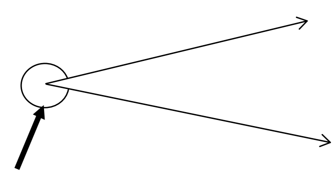
- **Arms of the angle:** the two lines that are at an angle to each other
- **The vertex:** the point where the two arms meet
- **Vertices:** plural of 'vertex'



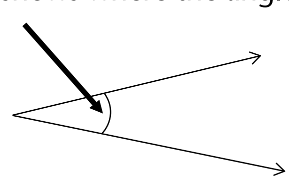
Symbols to describe angles:

Arrowheads on the lines mean that the lines keep on going. The length of an angle's arms does not change the size of the angle. Whether the arms are long or short, the angle size stays the same.

There are **two angles at a vertex** so it is important to show which one we are talking about.

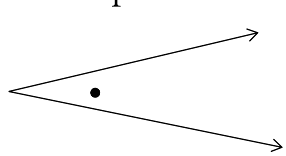


The arc shows where the angle is

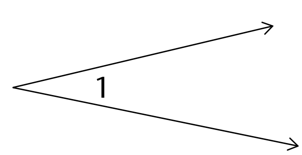


Labelling angles:

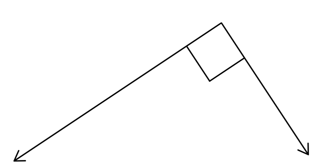
There are many different ways to label angles. Look at the examples below:



Using a dot or a star

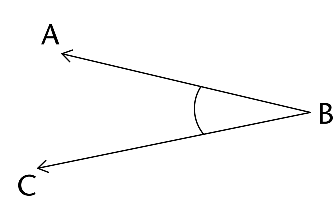


Angle 1



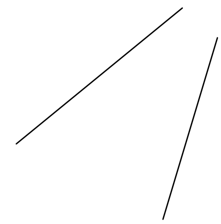
Right angle (90°)

You can name the angle on the right in different ways: you can say $A\hat{B}C$ or $C\hat{B}A$ or just $\hat{B}$. The 'hat' on the letter shows where the angle is.



REVISION: SEEING ANGLES AND DESCRIBING ANGLES

1. Look at the drawing on the right.



   (a) Are these lines at an angle to each other? ......

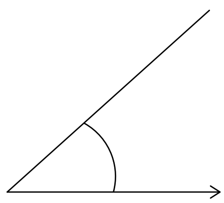
   Do the lines have to meet to be at an angle? ......

   (b) Use a pencil and your ruler to draw the lines a bit longer so they meet. Did you change the angle between the lines when you extended them? ......

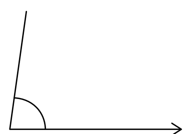
2. Arrange the angles from biggest to smallest. Just write the letters (a) to (f) in the correct order.

   ....

(a)

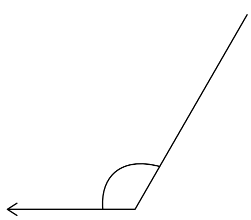


(b)

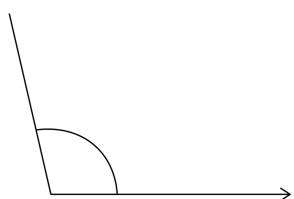


(c)

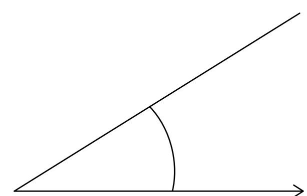
(d)



(e)



(f)

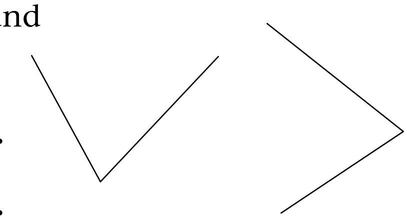


3. How can you check that an angle is a right angle without using any special mathematics equipment? (*Hint*: Think about where you can find right angles around you.)

   ....

   ....

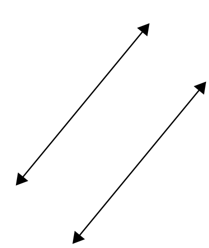
4. Are these two angles the same size? Describe how you found your answer. (*Hint*: A piece of scrap paper may help!)



   ....

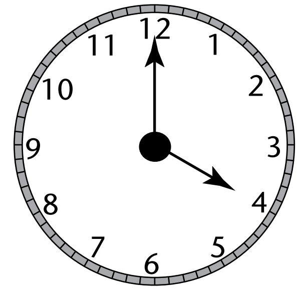
   ....

5. Two lines are drawn by holding down a ruler and drawing lines on both sides. What can you say about the two lines?



   ....

6. Look at the analogue clock face on the right. The minute hand and the hour hand make an angle. Focus on the smaller angle for now.
   (a) Explain why the angle between the hands at 8 o' clock is the same size as the angle at 4 o' clock.



   ....

   ....

   (b) Compare the angle at 2 o' clock with the angle at 4 o' clock. What do you notice? Why is this so?

   ....

   ....

   (c) Is the angle at 3 o' clock the same as the angle at a quarter past 12? Explain.

   ....

   ....

7. When you open the cover of a hardcover book you can make different angles. Can you think of at least five other situations in everyday life where objects are turned through angles? Say what the arms and the vertices are in each of your examples.

   ....

   ....

   ....

   ....

   ....

4.2 The degree: a unit for measuring angles

Imagine if we didn't have units for measuring length.

> *How would tailors make clothes to the right size without a tape measure? How could an architect design a safe and beautiful house without a ruler? How could we lay out a professional soccer field without being able to measure accurately in metres?*

We need units and measuring instruments in many situations. You know that we use metres, centimetres, kilometres, millimetres, etc. for measuring lengths.

We should also have units for measuring angles. The units we use for measuring angles are very ancient. No one today is completely sure why, but our ancestors decided many thousands of years ago that a revolution should be divided into 360 equal parts. We call these parts degrees. The symbol for a degree is °.

SOME FAMILIAR ANGLES IN DEGREES

1. Complete the table by filling in the size of each angle described.

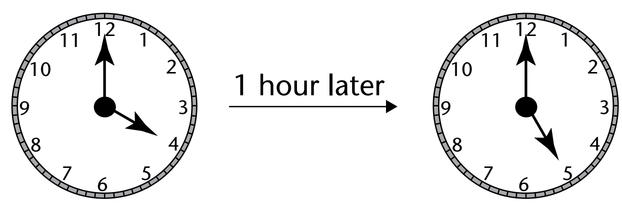
Angle (in words)	Angle (degrees)
right angle	90°
straight angle	
revolution	360°
half a right angle	
a third of a right angle	
a quarter of a right angle	22,5°
half a straight angle	
three quarters of a revolution	
a third of a revolution	

2. Look at the clock shown. How many degrees does:
   (a) the minute hand move in an hour?

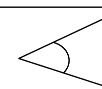
   ....

   (b) the hour hand move in an hour?

   ....



3. In Grade 6 you learnt that angles are classified into types. Complete the table. The first one has been done as an example for you.

Angle	Size of the angle	Sketch of the angle
Acute angle	Between 0° and 90°	
Right angle		
Obtuse angle		
Straight angle		
Reflex angle		
Revolution		

COMPARING ANGLES USING A4 PAPER

You need a sheet of A4 paper. At the corners you have four right angles. Number them and tear the corners off as shown in the diagram. Do not make them too small.

Now use your right angles to investigate the following situations:

1. Show that a straight angle is two right angles.
   You can sketch what you have done here.

2. Show that a revolution is four right angles.
   You can sketch what you have done here.

3. Create a right angle using three of your corners.
   You can sketch what you have done here.

4. Describe how you can use one of your corners to check if an angle is acute, right or obtuse.

   ....

   ....

   ....

   ....

5. (a) Fold corner 1 so that you can use it to measure 45°.
   (b) Fold corner 2 so that you can use it to measure 30°.
   (c) Fold corner 3 so that you can measure 22,5°.
   (d) Which is bigger: a right angle *or* half a right angle + a third of a right angle + a quarter of a right angle? Can you do a calculation to show that?

   ....

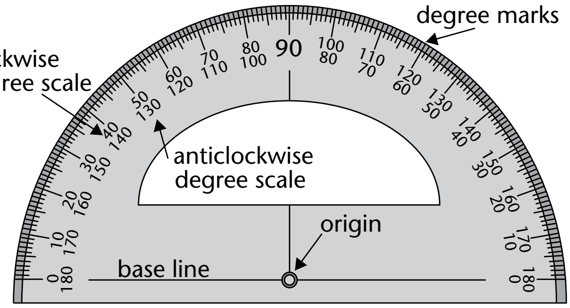
   ....

Important: Keep your folded pieces of paper for the next lesson!

4.3 Using the protractor

We have a special instrument for measuring angles. It is called a **protractor**. Look at the picture of a typical protractor with its important parts labelled.

Protractors can be big or small but they all measure angles in exactly the same way. The size of the protractor makes no difference to an angle's size.



MEASURING SOME FAMILIAR ANGLES

You need the four folded angles from the previous activity. If you didn't do that activity, then go back now and follow the instructions in question 5.

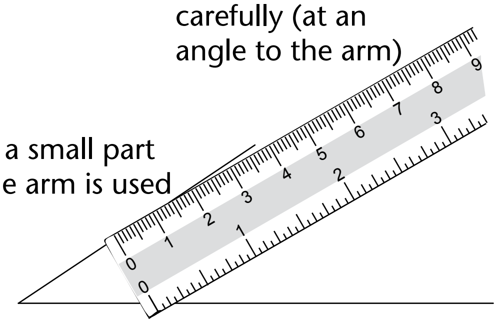
1. In a group of three or four, use your protractor to measure the angles that you made: 90°; 45°; 30° and 22,5°.
2. Did you measure the correct sized angle?

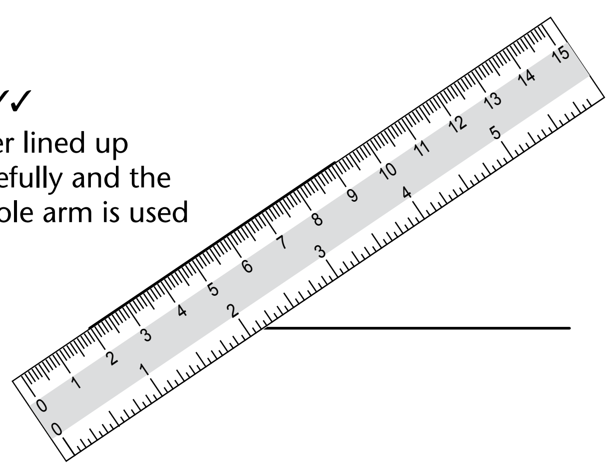
   If not, then ask yourself the following questions:
   - Did you put the vertex of the angle at the origin of the protractor?
   - Is the bottom arm of your angle lined up with the base line?
   - Did you fold your corners correctly?

HOW TO USE A PROTRACTOR TO MEASURE AN ANGLE

Step 1: Are the angle arms long enough?

The angle arms must be a little longer than the distance from the origin of the protractor to its edge. If they are too short, use a sharp pencil and a ruler to make them longer. Be careful to line the ruler up with the arm.





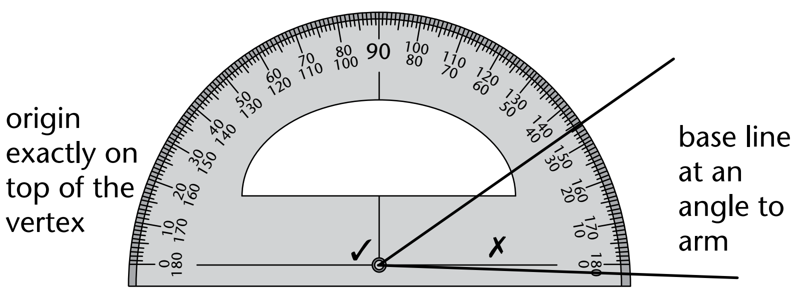
Now you are ready to start measuring your angle.

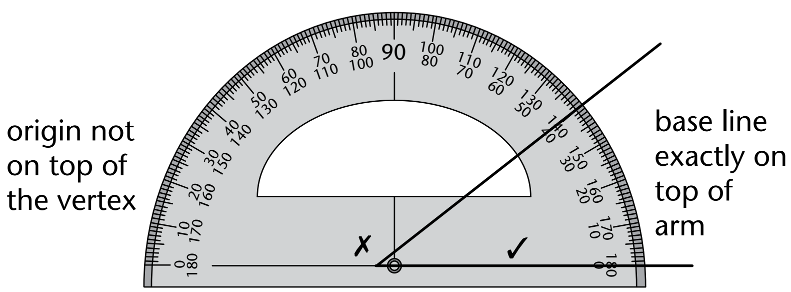
Step 2: Line up the angle and your protractor

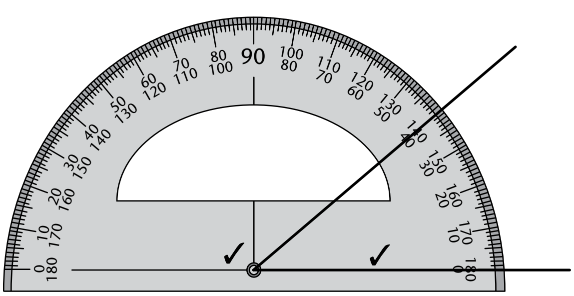
Place your protractor on top of the angle. Make sure of the following:

- the origin is exactly on the vertex of the angle, and
- the base line is exactly on top of one of the arms of the angle.

Keep adjusting the position of the protractor until the origin and the base line are exactly lined up.



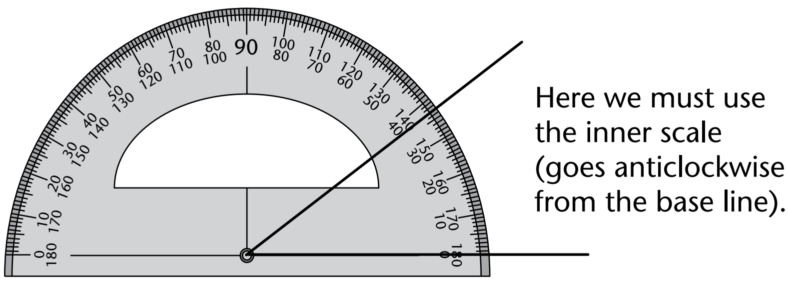




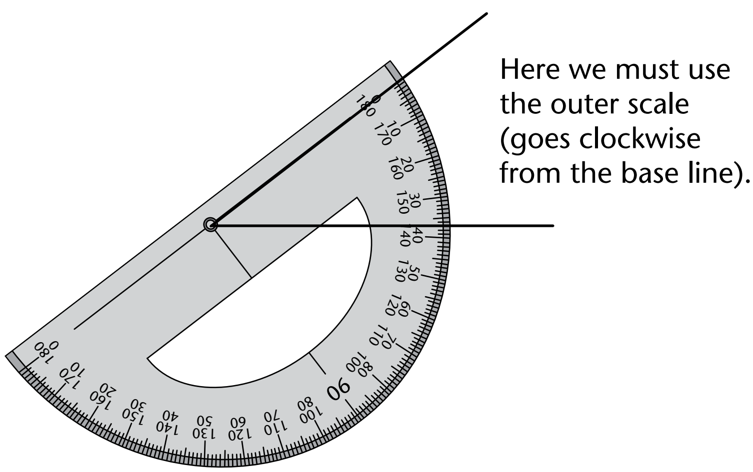
Once your protractor is in the correct place keep a finger on the protractor to stop it from moving. If it moves… start again! You are now ready to make a measurement.

Step 3: Measure the angle

A protractor gives a clockwise degree scale and an anticlockwise degree scale. You choose the correct scale by finding the one that starts with 0° on the angle arm. Look at where the other angle arm passes under the degree scale. That is where your measurement is.



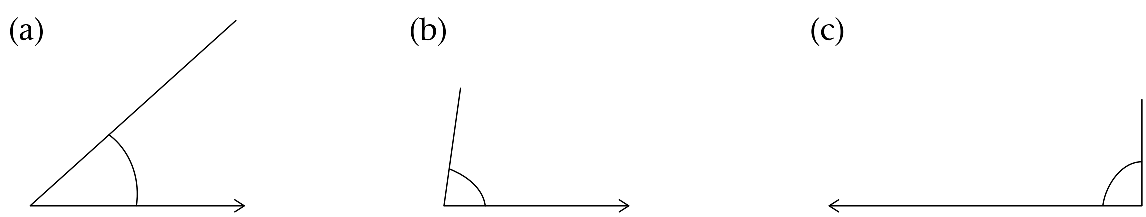
You can also place the protractor on the angle using the other arm. Then the correct position looks like this:



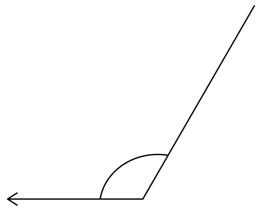
The angle in the pictures above is 37°. Do you agree? Do you see that there are two ways to measure an angle?

PRACTISE MEASURING WITH A PROTRACTOR

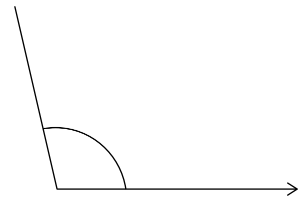
1. Measure the angles and complete the table on the next page. You can extend the arms if you need to; it doesn't matter if they go across text or another drawing.



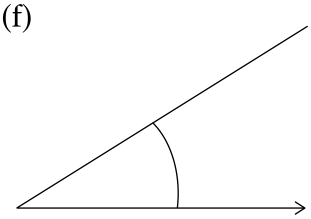
(d)



(e)



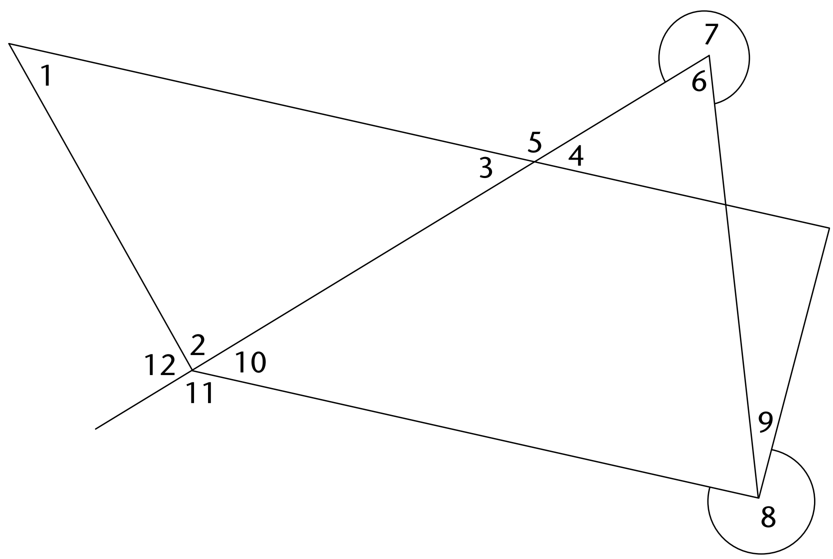
(f)



Angle	(a)	(b)	(c)	(d)	(e)	(f)
Angle size in degrees						

2. Measure all the numbered angles in the following figure. Some angles can be measured directly, others not. Your protractor cannot measure reflex angles like angles 7 and 8. So you will have to make a plan!

Angle	**Size**
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	



3. Write a short note for yourself about measuring reflex angles here:

....

....

....

SOME THINGS TO THINK ABOUT

Look at your answers in question 2.

1. How do angles 3 and 4 compare?

....

2. What about angles 6 and 7?

....

3. What about angles 4 and 5? ....

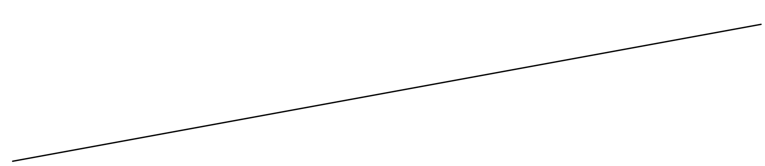
4. There are some interesting ideas here. Try to do some further investigation and show your teacher what you discover.

4.4 Using a protractor to construct angles

CONSTRUCTING ANGLES TO A GIVEN LINE

Work together with a partner on this activity. You need your protractor, a sharp pencil and a straight ruler.

1. Your first challenge is to construct a line at exactly right angles to the one below. Begin by choosing a point on the line. You must mark this point clearly and neatly with a small dot. Then use your understanding of a protractor to draw a 90° angle.



2. Now fill in the missing words in the steps:

   Step 1: Choose a point anywhere on the line. Make a small mark on the line. (You don't always have a choice here. Sometimes you must use a specific point on the line.)

   Step 2: Place the protractor with its .... on the line and its origin exactly on top of the ....

   Step 3: Make a small, clear mark at the ....

   Step 4: Use a ruler to line up the two .... and draw a straight line that passes exactly through them.

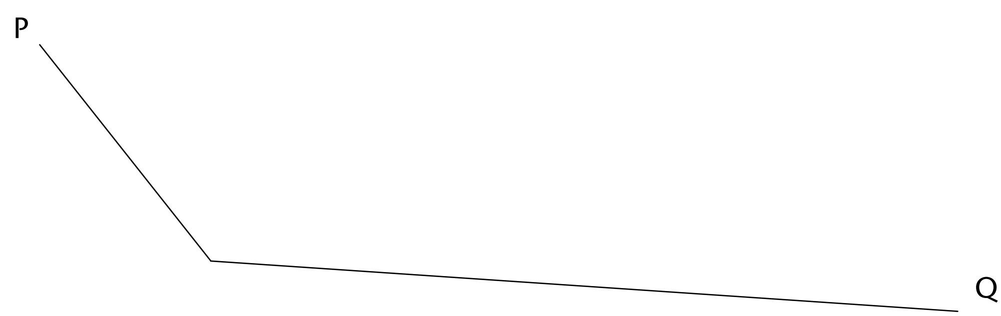
3. Construct the angles using the line below. The line below will be one arm of the angles you are going to construct. The vertex for each of your angles is the point labelled O where the tiny vertical line cuts the long horizontal one. Your angles must be measured *anticlockwise* from the line.

   (a) 23° (b) 45° (c) 65° (d) 79° (e) 90°
   (f) 121° (g) 154° (h) 180° (i) 200° (j) 270°
   (k) 300°

Angle direction
The line you have been given below is called a **reference line**.
Mathematicians usually measure angles **anticlock-wise** from the reference line.

4. Use the line below. At each end you need to draw lines at an angle of 60° to form a triangle. What sort of triangle is this? ....

5. Complete the quadrilateral below. The angle at P must be 52° and the one at Q, 23°.



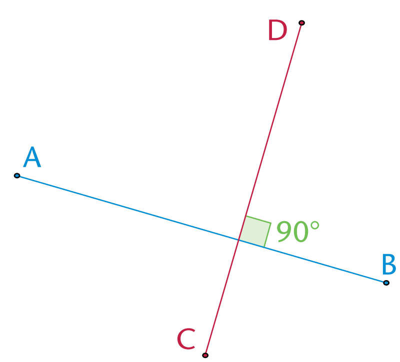
4.5 Parallel and perpendicular lines

Perpendicular lines meet each other at an angle of 90°.

The sketch shows two perpendicular lines.

We say: AB is perpendicular to DC.

We write: AB $\perp$ DC



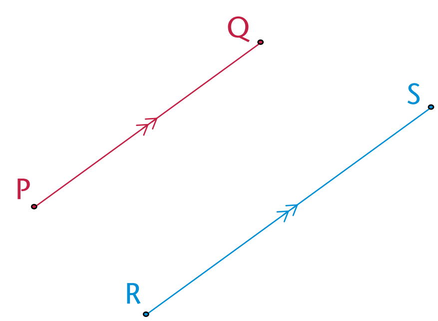
Parallel lines never meet each other. They are an equal distance apart. They have the same direction.

The sketch shows two parallel lines.

We say: PQ is parallel to RS.

We write: PQ || RS

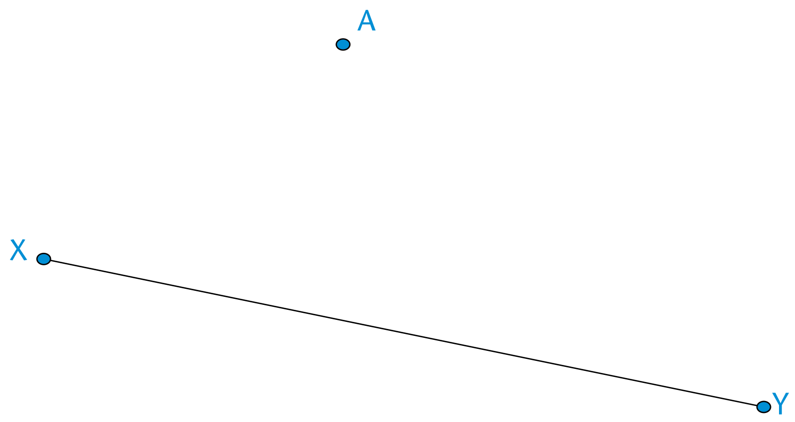
The arrows on the middle of the lines show that the lines are parallel to each other.



CONSTRUCTING PERPENDICULAR AND PARALLEL LINES

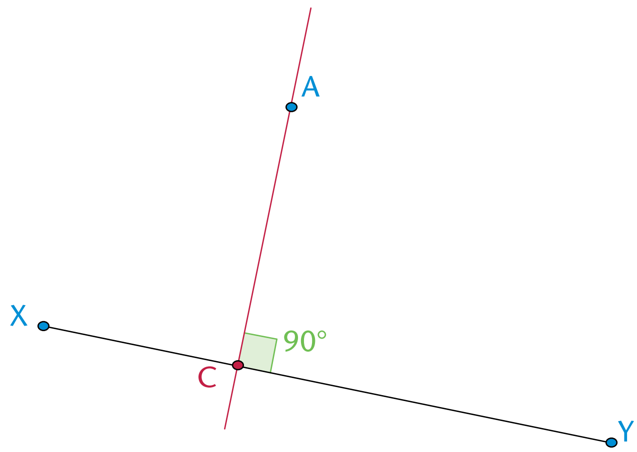
When constructing parallel lines, remember that the lines always stay the same distance apart. Follow the steps below to draw perpendicular and parallel lines using a protractor and a ruler.

1. We want to draw a line that is parallel to XY and that passes through point A.



Step 1: Draw a perpendicular line between A and XY.

Use your protractor to draw a line that goes through A and is at 90° to XY.
Label the point C where your new line touches XY.
Look at the sketch below if you get stuck.



Step 2: Measure the perpendicular distance between the point and the line.

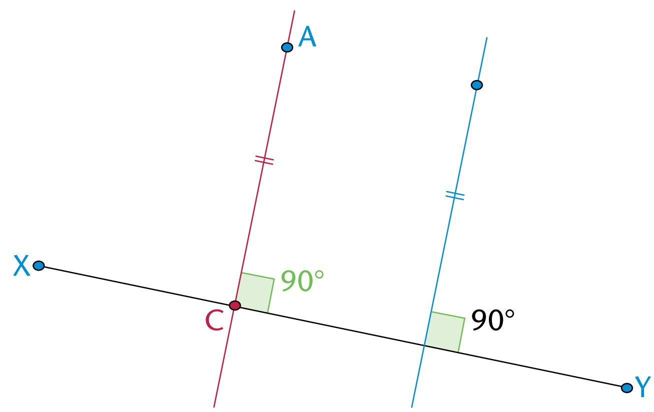
Write down the length of AC: ....

Step 3: Draw a point that is the same distance from the line.

Draw another line that is perpendicular to line XY.

Mark off the same length as AC on that line.

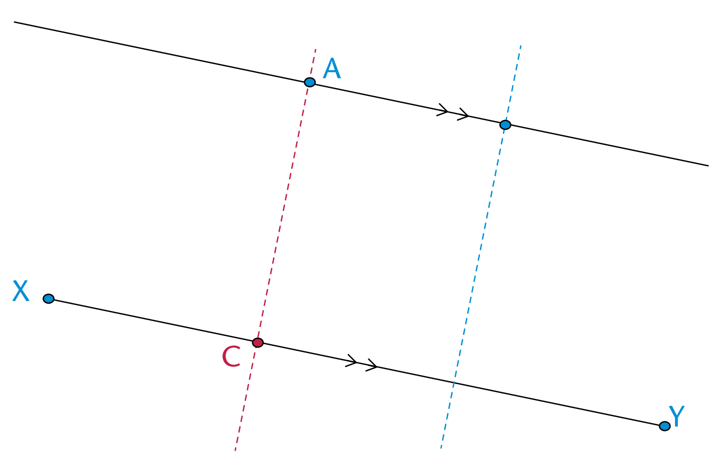
The sketch shows what you must do.



Step 4: Draw the parallel line.

Join A with the new point that is an equal distance away from XY.

You now have a parallel line.



2. In your exercise book, practise constructing perpendicular and parallel lines using a protractor and a ruler.

4.6 Circles are very special figures

And now for something slightly different … let us have a look at **circles**.

A CIRCLE WITH STRING

You may need to work with a partner here. You need two sharp pencils and a short length of string, an A4 sheet of paper and a ruler.

1. Tie the string to both pencils with double knots. The knots must be firm but not tight. The string must swing easily around the pencils without falling off. Once you have tied your string, the distance between the pencils when the string is tight should not be more than 8 cm.
2. Your partner must hold one pencil *vertically* with its point near the centre of the sheet of paper.
3. Now carefully move the tip of the other pencil around the middle one, drawing as you go. Try to keep the string *stretched* and the pencil *vertical* as you draw.

   If you have been careful, you have a circle (well, hopefully something pretty close to a circle). You can swop now so your partner also has a turn drawing while you hold the centre pencil.
4. Mark three points on the circle line. Measure the distance between the point and the centre of the circle for each. If you have a circle you should find that the distances are the same.

Circles are special for many reasons. The most important reason is the following:

The distance from the centre of a circle to the edge is the same in any direction.

This distance is called the **radius**.

We pronounce this: "ray-dee-us".

The plural of **radius** is **radii**.

We pronounce this: "ray-dee-eye".

Think about it

Can you think of any other figure where the distance between the centre and the edge is constant in all directions?

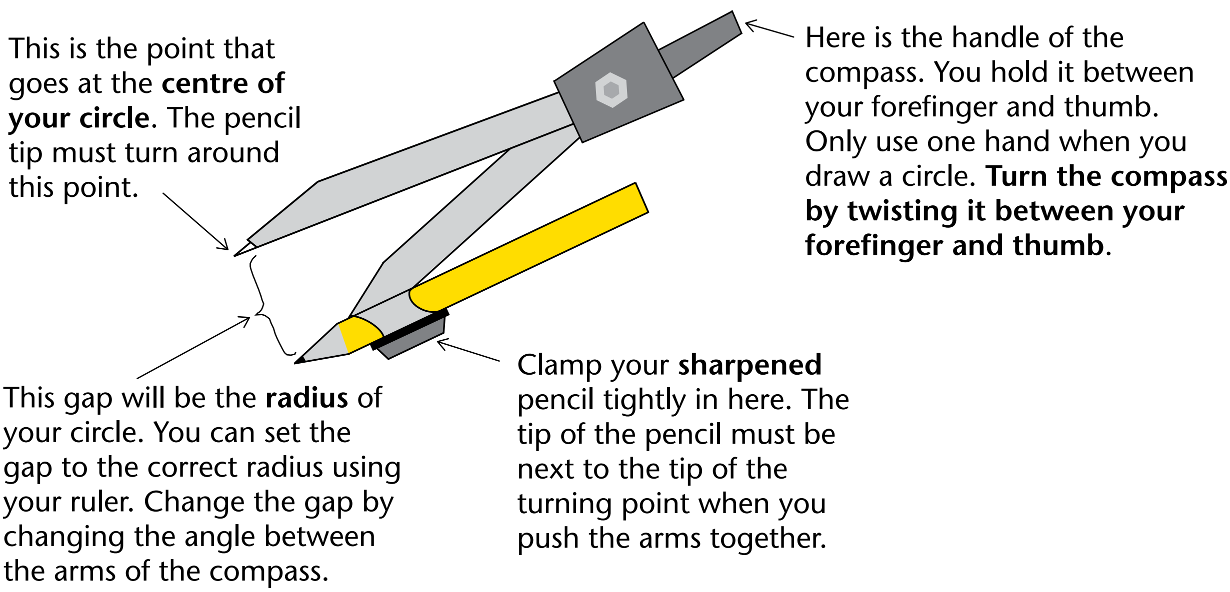
- A square?
- A hexagon?
- What about an oval shape (ellipse)?

Do some investigation to see what you can find.

Do you agree that the two pencils and string are not a good way to draw circles? The string is stretchy. It is difficult to change the radius. And, the drawing pencil can wander off course and make a spiral or a wobbly curve. We need something better.

4.7 Using the compass

We need a special instrument for drawing circles. It must have a pointy tip, like the centre pencil. It must also have a drawing tip, like the pencil you moved. If you can set the distance between these two tips, you can draw circles of any radius. This instrument is called a **pair of compasses**, or often just a **compass**.

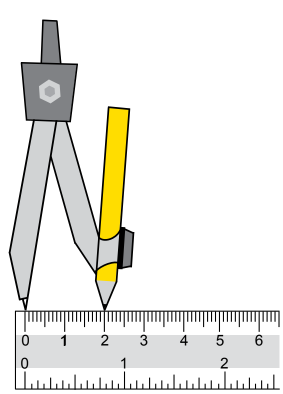


CONSTRUCTING CIRCLES WITH A COMPASS

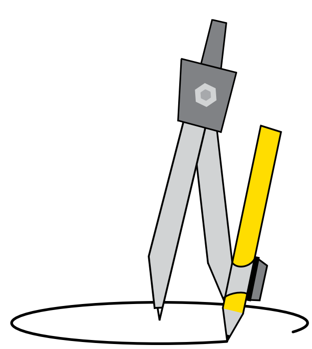
1. At the top of the next page you will see a point labelled A. Follow the steps below and on the next page to draw a circle with a radius of 2 cm. The centre must be at A.

   Step 1: Place the pointed tip on the zero line of your ruler. Carefully widen the angle between the arms. Move the pencil tip until it is exactly at 2 cm. Make sure that the pointed tip is still on zero. Be careful not to change the gap once it is set to 2 cm.

   Step 2: Gently push the pointed tip into point A. Push just deep enough into the paper to keep it in place. This will be the centre of your circle.



Step 1



Step 4

• A

Step 3: Hold the handle between the forefinger and thumb of your writing hand. Keep your other hand out of the way. Use only one hand when you draw a circle with a compass.

Step 4: Twist the handle between your thumb and forefinger. If you are right-handed it is easiest to twist the compass clockwise. If you are left-handed, turn anticlockwise. Let the pencil tip *drag* over the paper. Don't push down too hard on the pencil. Rather, push down lightly on the pointed arm as you draw. The pencil tip must move smoothly and easily.

2. Draw concentric circles at centre A above with radii of 3 cm, 4 cm, 5 cm and 6 cm. Set the gap carefully each time. Write the radius on the edge of each circle.

Concentric circles have the same midpoint.

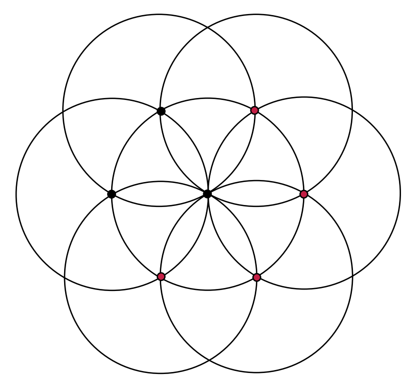
Learning to use a compass is like learning to ride a bicycle. It takes co-ordination and practice. Don't be embarrassed if it goes wrong. With practice you will get very good at it. If your circles end up being all wobbly lines, just begin again!

Here are some tips for drawing circles:

- If your circles are turning into spirals it is because the arms of your compass have moved. Check their width again against a ruler.
- If the arms of your compass won't stay in the position you set them at, it is because the nut at the hinge below the handle is loose. Ask your teacher to help you if you can't tighten it yourself.
- If you can't do the twist, imagine you have a small piece of soft clay between your thumb and forefinger and you are trying to roll it into a small strip. The twist for turning your compass uses the same type of sliding movement. Let the compass hang from your hand in the air and twist the handle. Then try it on scrap paper a few times until you can turn the compass easily.

CIRCLES ON CIRCLES

It's time to have some fun with the compass while getting better at using it. Follow these instructions to draw the beautiful pattern shown on the right in your exercise book.



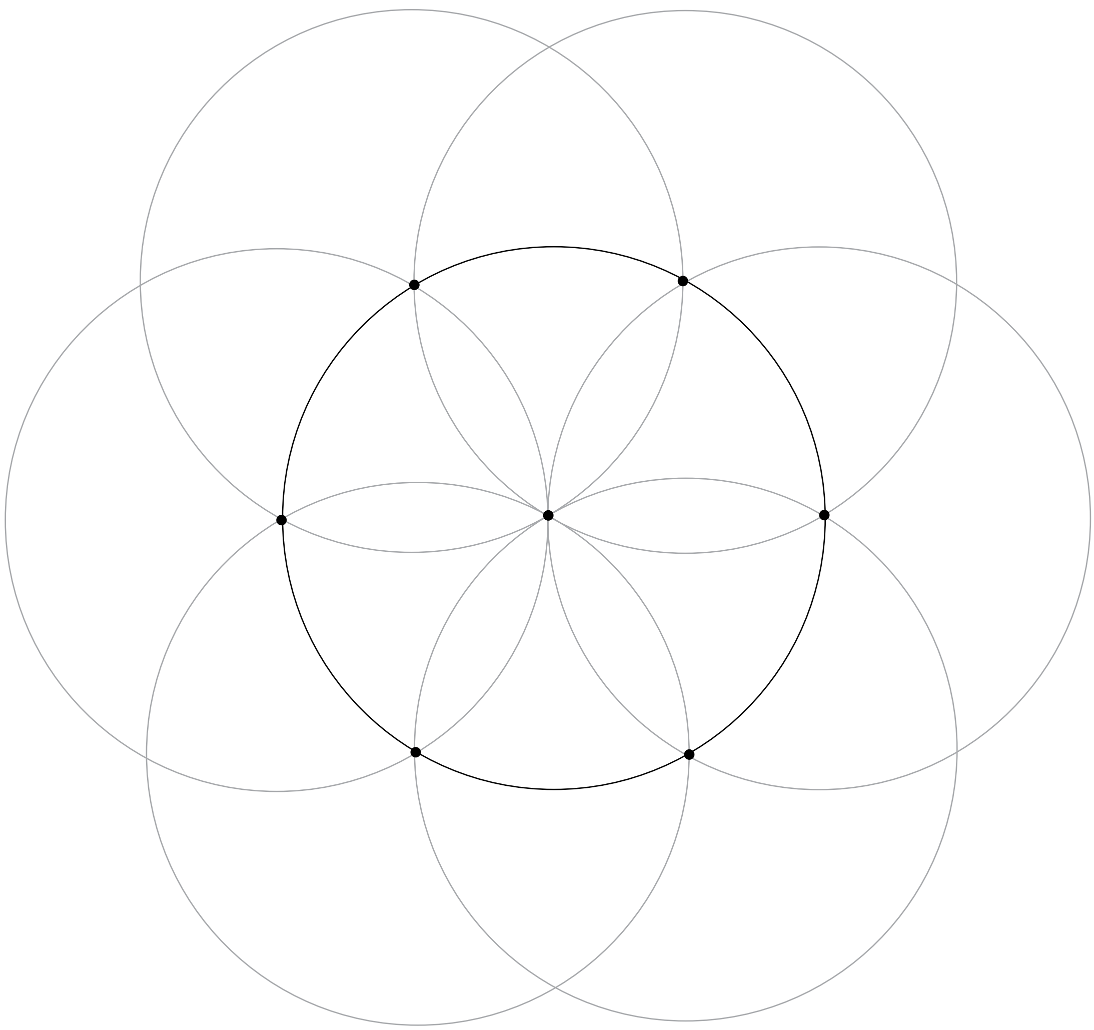
1. Make sure your pencil is sharp; then place it in the compass.
2. Set the radius to 4 cm. Draw a circle at the centre of your page. Important: your radius must stay the same for the whole activity.
3. Put your compass point anywhere on the circle edge. Draw another circle. This circle should pass through the centre of your first circle (they have the same radius).
4. Your second circle cuts the first circle at two points. Choose one of these points. Place your compass point at this point. Draw another circle of radius 4 cm.
5. Repeat step 3 with your third circle, fourth circle etc. You should end up with six circles on your first circle. That is, seven circles in total.
6. Decorate it as you please. (You can decorate your pattern further by adding more circles or joining points with straight lines, and so on. See what patterns and shapes you can discover among all the circles.)

4.8 Using circles to draw other figures

GEOMETRIC FIGURES HIDING IN THE CIRCLES

Below is a set of seven circles like the one you drew. Sit with a partner and try to find hidden polygons.

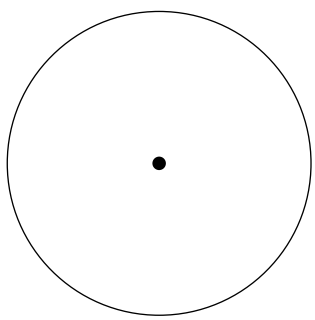
You will find these polygons by joining the points where the circles cut each other. The points will be the vertices of the polygons. Look carefully. There are triangles, quadrilaterals, pentagons and hexagons. When you can see them, neatly and carefully rule in their sides with a pencil. If there is not enough space on the set of circles below, redraw the circles on a separate piece of paper and show the figures there. If you wish, you can measure the angles at each vertex and the lengths of the sides.



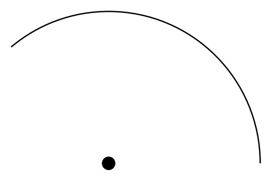
Arcs of circles

We do not have to draw whole circles to construct figures. We are only really interested in the points where the circles cross each other, so we could just draw arcs where they cross. Next year, you will use arcs in your geometric constructions.

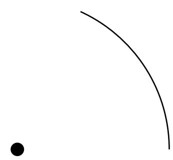
> An **arc** is a small part of a circle.
> We use the term **circumference** when we refer to the distance around a circle or around any other curved shape.



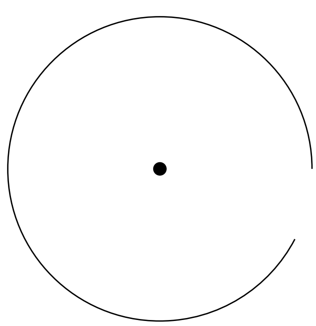
This is a full circle.



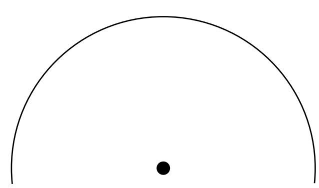
This is only part of a circle. A part of a circle like this is called an ***arc****.*



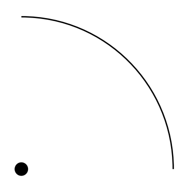
This arc is smaller.



This arc is almost a full circle.



This arc is just more than a semi-circle.



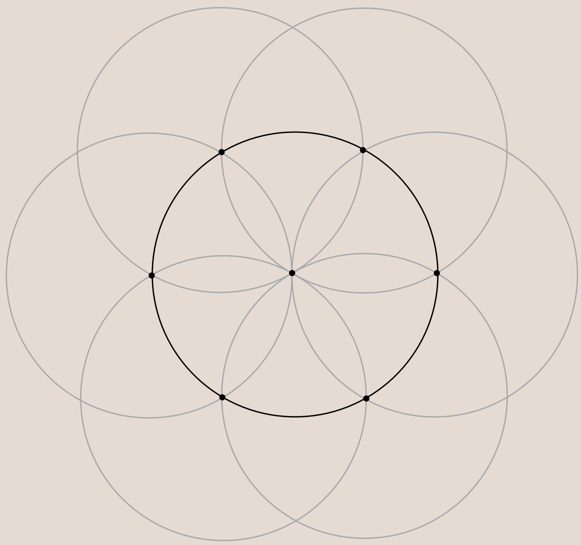
This arc is a quarter circle.

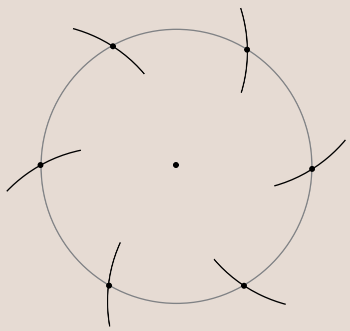
Do the following in your exercise book:

1. Draw an arc using a radius of 3 cm.
2. Draw an arc bigger than a quarter circle, using a radius of 5 cm.
3. Draw an arc smaller than a quarter circle, using a radius of 5 cm.

ENRICHMENT

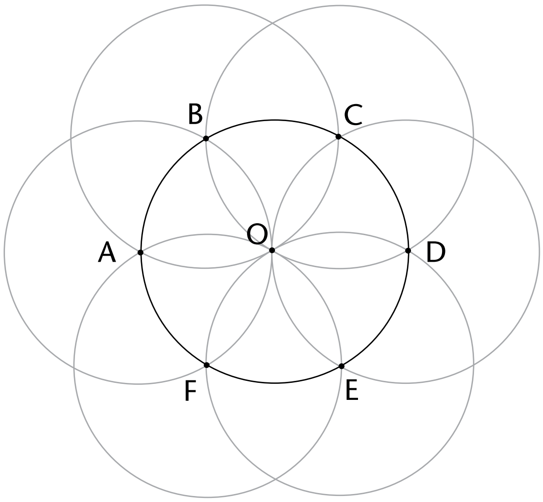
Once you have finished the work in section 4.8, experiment with drawing only the arcs that you need in various constructions. Here is an example to show how to construct a regular hexagon with only arcs:





FAMILIAR FIGURES IN THE SEVEN-CIRCLE PATTERN

For this activity you need five seven-circle sets like the ones drawn in the previous two activities. Start by drawing these on blank pieces of paper. Don't make your radius bigger than 4 cm. Number your sets figure 2 to figure 6. Label each figure as shown on the right.



1. Follow the instructions below.
    - Figure 1: Use the figure alongside. Draw lines connecting AB, BC, CD, ... up to FA.
    - Figure 2: Draw lines connecting A, O and B.
    - Figure 3: Draw lines connecting B, F and D.
    - Figure 4: Draw lines connecting BC, CE, EF and FB.
    - Figure 5: Draw lines connecting CD, DE, EF and FC.
    - Figure 6: Draw lines connecting AB, BC, CE and EA.

2. Complete the table below.

   It shows the name of each figure and its properties.

   Figure 1 (on the right) has been done as an example.

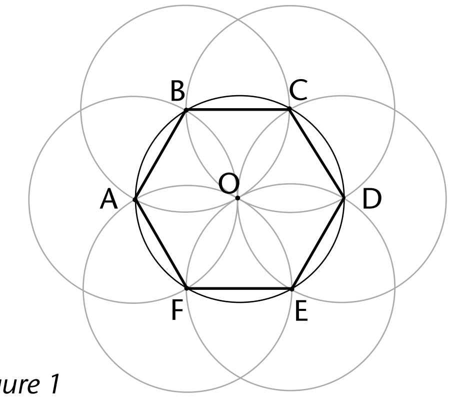


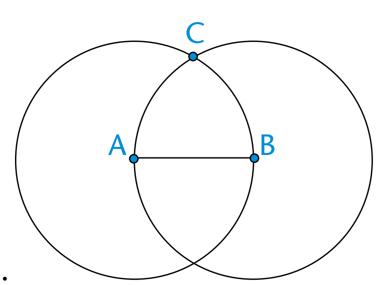
Figure 1

Figure	Name of figure	Properties
1	Regular hexagon	6-sided figure. All the sides are equal. All the interior angles are equal.
2		
3		
4		
5		
6		

CONSTRUCT SOME MORE FIGURES

Read the instructions carefully and follow them exactly.

1. (a) Draw a line in your exercise book. The line should be between 3 and 6 cm long. Draw it in the middle of your page.
   (b) Label the ends A and B.
   (c) Place the point of your compass at point A. Carefully set the radius of your compass to the distance between A and B.
   (d) Draw a circle with the compass point at A.
   (e) Draw another circle with the compass point at B without changing the radius width.
   (f) The circles cross at two points. Choose one of these points. Label it C. Check that you are on the right track by comparing your sketch to the one on the right.



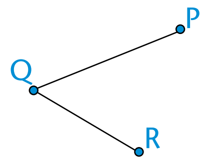
(g) Carefully rule the lines AC and BC.

(h) What sort of figure is ABC? Check this by measuring angles. Why do you think this happened?

....

....

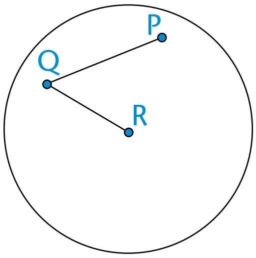
2. (a) Draw two lines PQ and QR in your exercise book.
   - The lines meet and form an angle at Q.
   - You can make your angle any size.
   - Make your line lengths different.
   - Do not make your lines longer than 6 cm each.



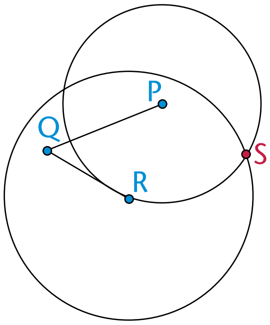
Step (a)

(b) Place your compass point at point Q. Set the radius of your compass to the distance QP. Place the compass point at R. Draw a circle.

(c) Place the compass point back at Q. Set the radius to the length QR. Place the compass point at P. Draw a circle.



Step (b)



Steps (c) and (d)

(d) The two circles cross at two points. Decide which point will be the vertex of a parallelogram. Call this point S.

(e) Join the lines SP and SR. Is PQRS a parallelogram?

....

Something to think about
Why does this method form a parallelogram?

4.9 Parallel and perpendicular lines with circles

PARALLEL AND PERPENDICULAR

1. Revision: Complete these definitions.

(a) When one line is parallel to another line, the lines ...

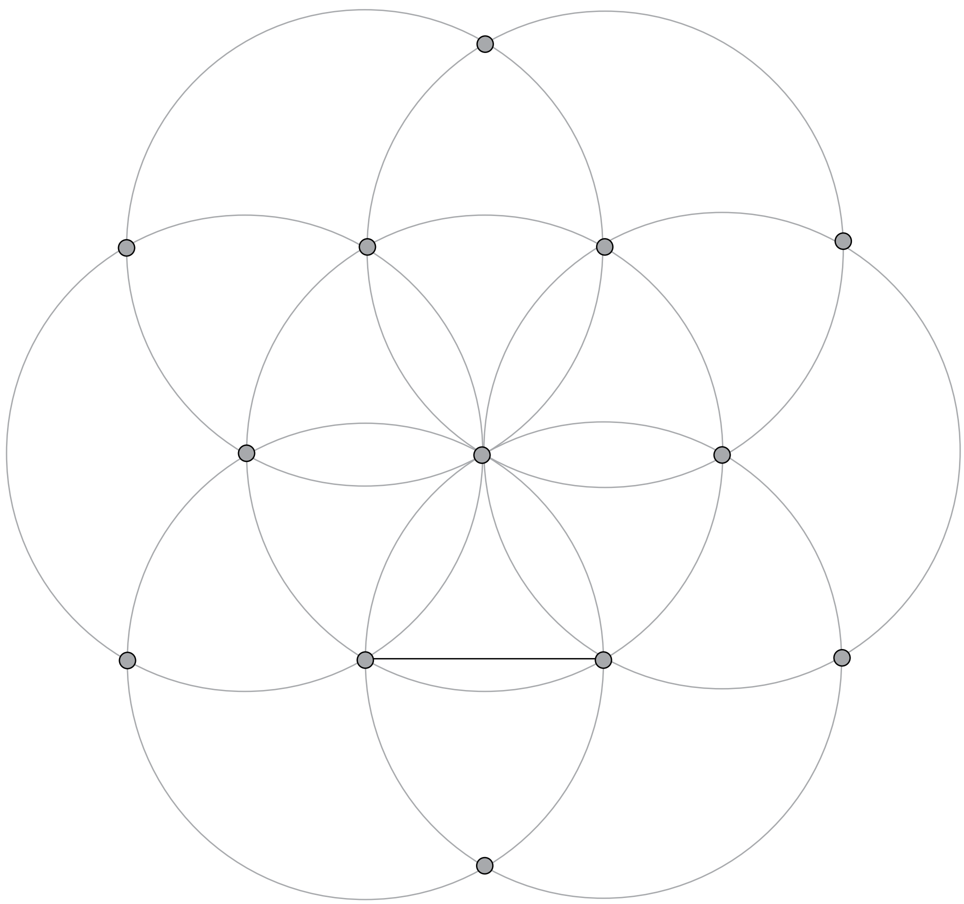
....

(b) When one line is perpendicular to another line, the lines ...

....

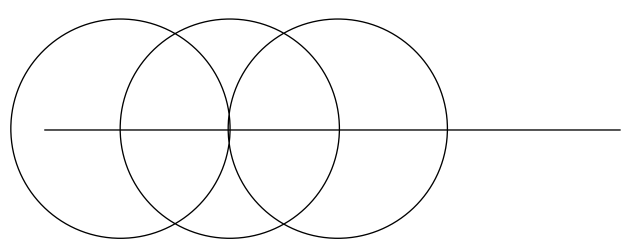
2. A seven-circle figure has been drawn below. The intersection points have been marked. A line segment has been drawn in. Use a ruler and pencil to join pairs of points so that the lines are:
   (a) parallel to the line segment
   (b) perpendicular to the line segment.

When two lines (or arcs) cross each other we say they **intersect**.
The **intersection point** is the place where they meet.



You should have drawn 7 lines (2 parallel and 5 perpendicular to the line segment). Compare your lines with a friend's lines. Do you agree?

3. In your exercise book, draw a few circles with the same radius along a line. Start by drawing a line. Then use your compass to draw a circle with the midpoint on the line.



Keep your compass the same width and draw another circle with the centre where the first circle crossed the line. Repeat as many times as you wish. In the example at the bottom of the previous page only three circles have been drawn.

(a) Can you find that example in the seven-circle figure? Look carefully until you see it.

(b) Can you see where you can construct lines that are perpendicular to the given line? Draw them carefully with a pencil and your ruler.

(c) Can you see the two lines that are parallel to the given line? Draw them in too.

4. Use circles to construct a line that is perpendicular to the line below.

5. Use circles to construct a line that is parallel to the line below.

EXTENSION

1. Set your compass at a certain distance, for example 3 cm, and investigate points that are the same distance from a fixed point, P.

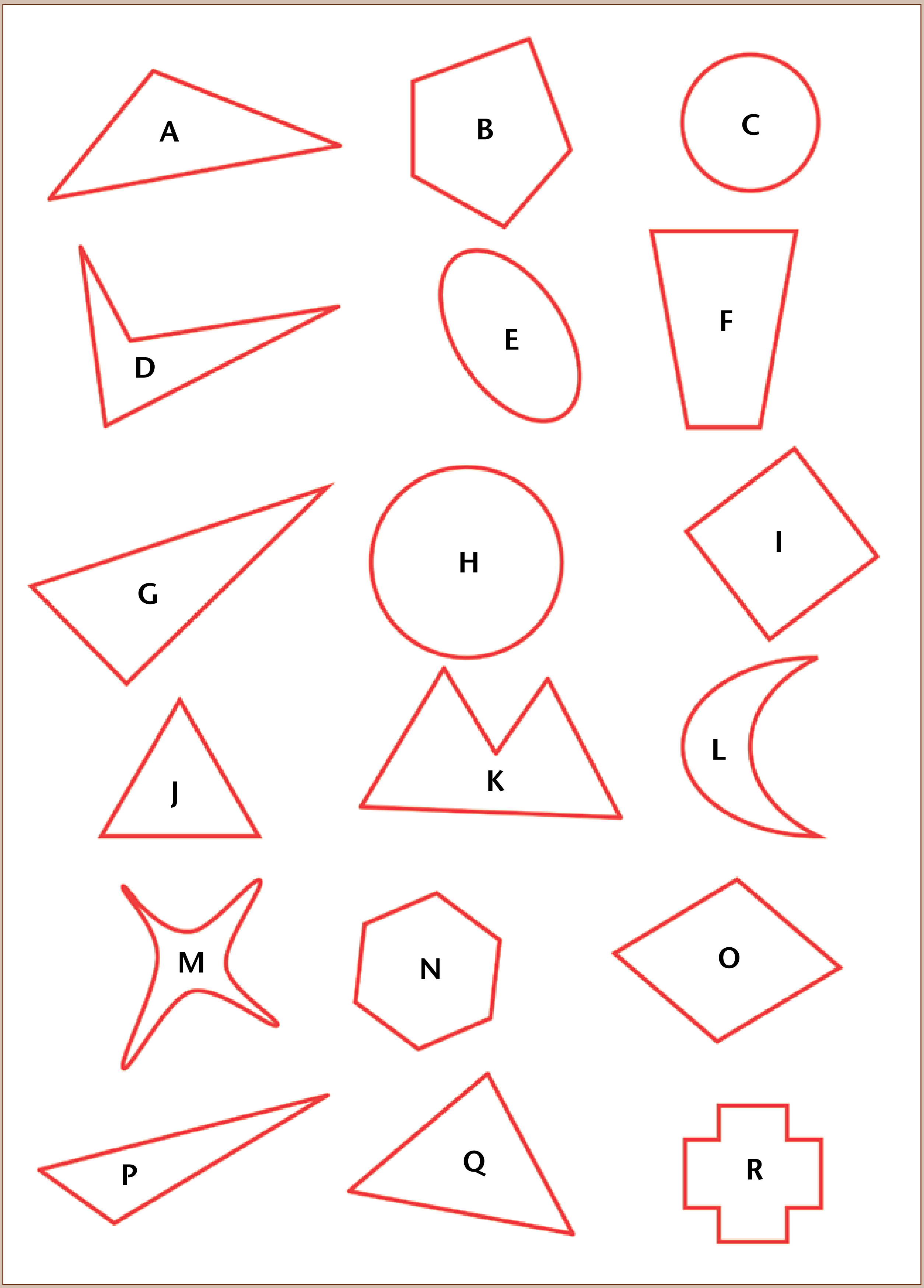
• P

2. Use your compass and investigate all the points that are at the same distance, for example 3 cm, from two fixed points, A and B.

• A　　　　　　• B

Chapter 5
Geometry of 2D shapes

In this chapter, you will learn about different kinds of 2D shapes. You will learn the names given to different shapes. You will also learn about the different properties that different types of shapes have in relation to their sides and angles.


A
B
C
D
E
F
G
H
I
J
K
L
M
N
O
P
Q
R

5 Geometry of 2D shapes

5.1 Triangles, quadrilaterals, circles and others

DECIDE WHICH IS WHICH AND DRAW SOME FIGURES

A **triangle** is a closed figure with three straight sides and three angles.

A **quadrilateral** has four straight sides and four angles.

A **circle** is round and the edge is always at the same distance from the centre.

1. Which shapes on the opposite page are circles? ....
2. Which shapes on the opposite page are triangles? ....
3. Which shapes on the opposite page are quadrilaterals? ....

Use your ruler to do the following:

4. Make a drawing of one triangle with three acute angles, and another triangle with one obtuse angle.

5. (a) Draw a quadrilateral with two obtuse angles.

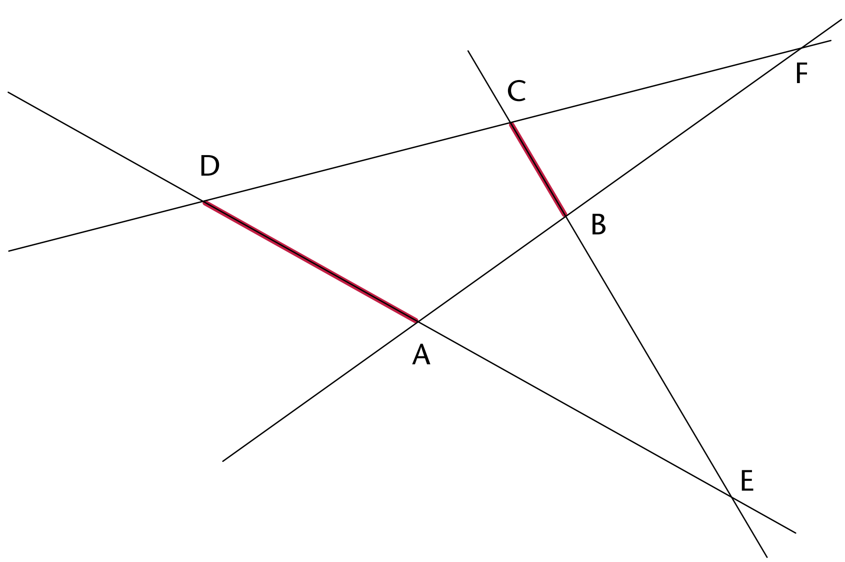
   (b) Can you draw a triangle with two obtuse angles? ....

6. (a) Draw a triangle with one right angle, and a triangle without any right angles.

(b) Can you draw a triangle with two right angles?

(c) Can you draw a quadrilateral with four right angles?

7. These four lines form quadrilateral ABCD.



The two red sides, BC and AD, are called **opposite sides** of quadrilateral ABCD. Which other two sides of ABCD are also opposite sides?

....

8. The lines DA and AB in the figure in question 7 are called **adjacent sides**. They meet at a point that is one of the vertices (corner points) of the quadrilateral.
   (a) Name another two adjacent sides in ABCD.

   ....

   (b) AB is adjacent to DA in the quadrilateral ABCD.
   Which other side of ABCD is also adjacent to DA? ....

9. William says:
   "Each side of a quadrilateral has two adjacent sides.
   Each side of a quadrilateral also has two opposite sides."
   Is William correct? Give reasons for your answer.

   ....

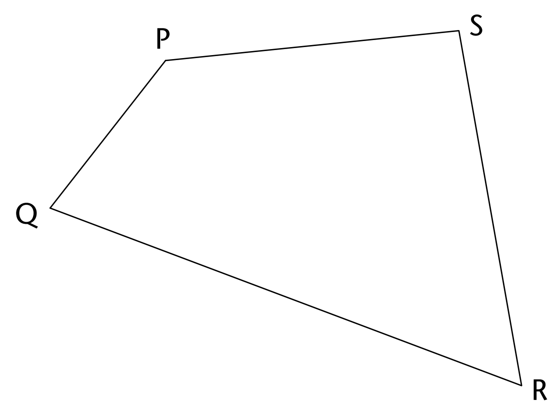
   ....

10. William also says:
    "In a triangle, each side is adjacent to all the other sides."
    Is this true? Give a reason for your answer.

    ....

    ....

11. In each case, say whether the two sides are opposite sides or adjacent sides of the quadrilateral PQRS.



    (a) QP and PS

    ....

    (b) QP and SR

    ....

    (c) PQ and RQ

    ....

    (d) PS and QR

    ....

    (e) SR and QR

    ....

5.2 Different types of triangles

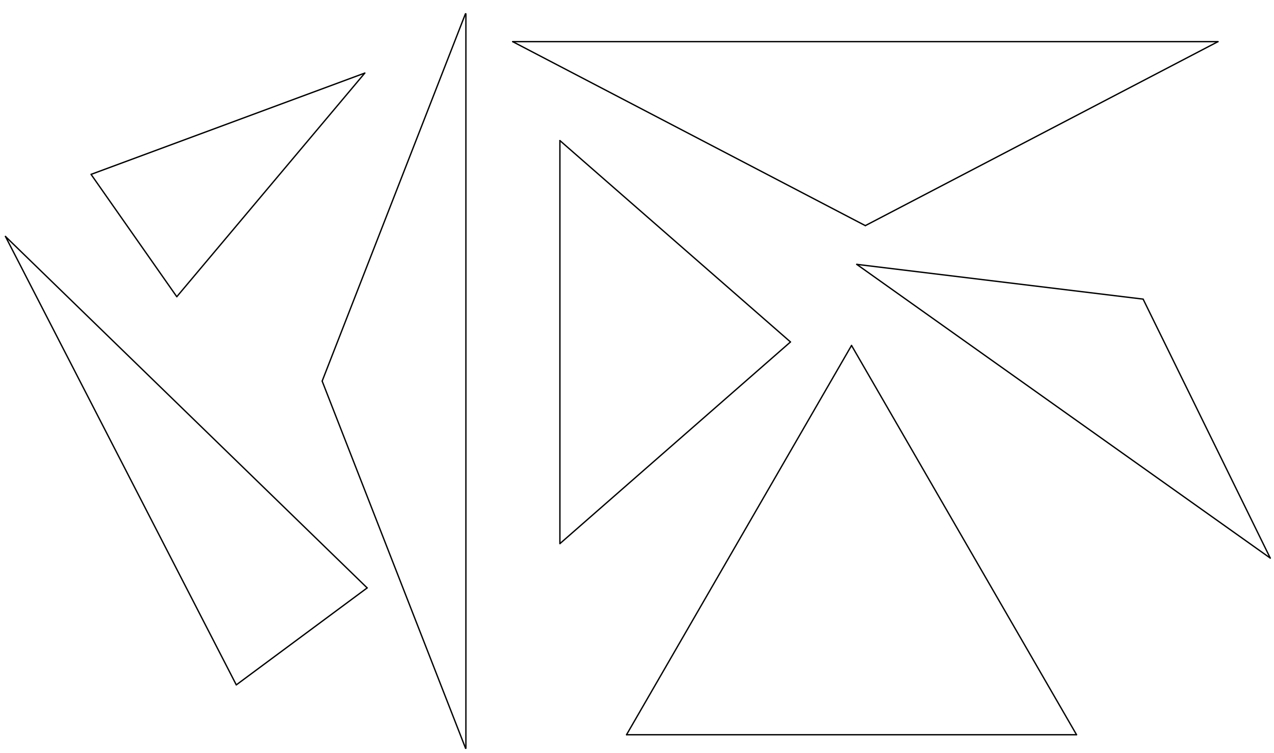
EQUILATERAL, ISOSCELES, SCALENE AND RIGHT-ANGLED TRIANGLES

A triangle with two equal sides is called an **isosceles triangle**.

A triangle with three equal sides is called an **equilateral triangle**.

A triangle with a right angle is called a **right-angled triangle**.

A triangle with three sides with different lengths and no right angle is called a **scalene triangle**.



1. Measure every angle in each of the **isosceles triangles** given above. Do you notice anything special? If you are not sure, draw more isosceles triangles in your exercise book.

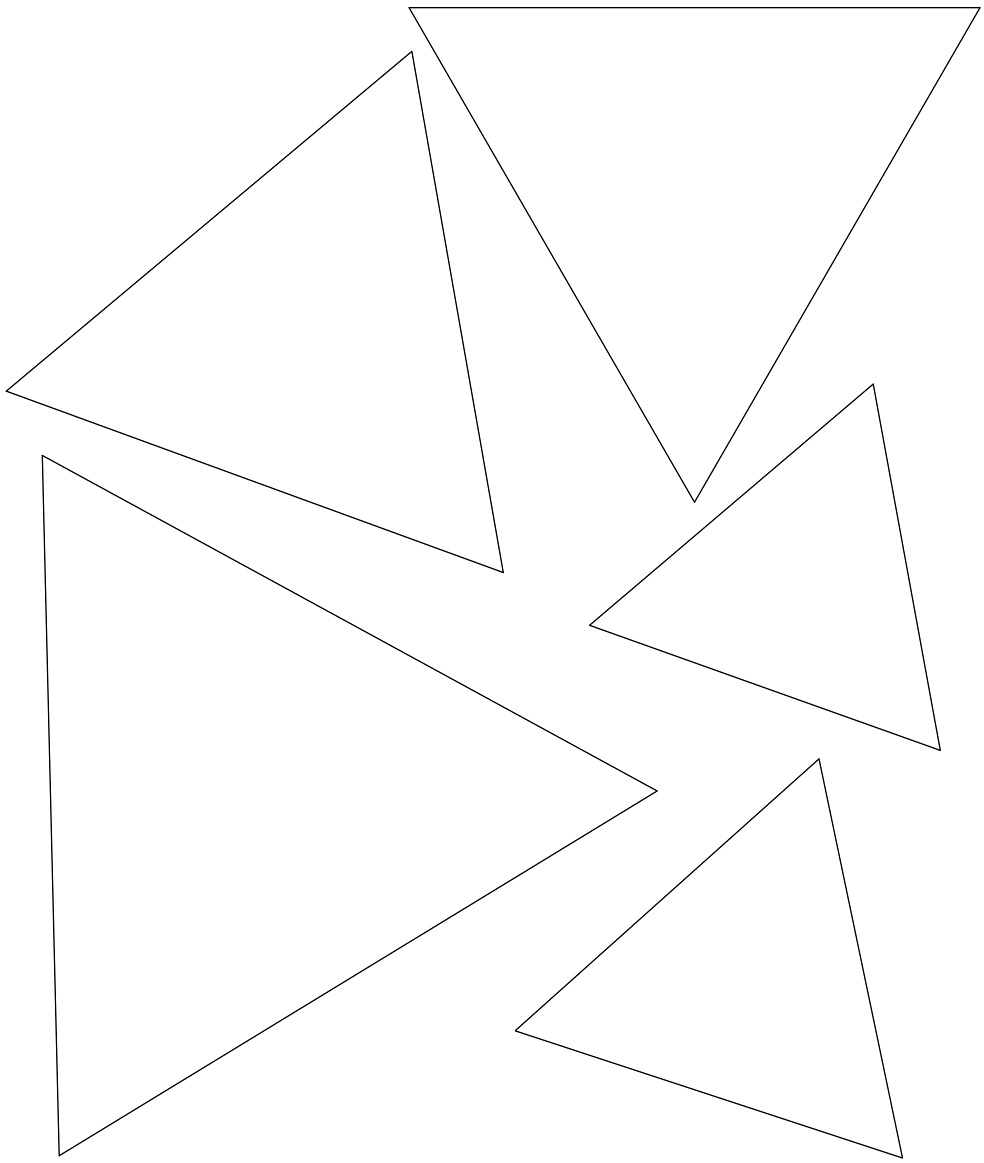
....

....

2. Measure the angles and sides of the following triangles. What is special about these triangles? In other words, what makes these triangles different to other triangles?

....

....

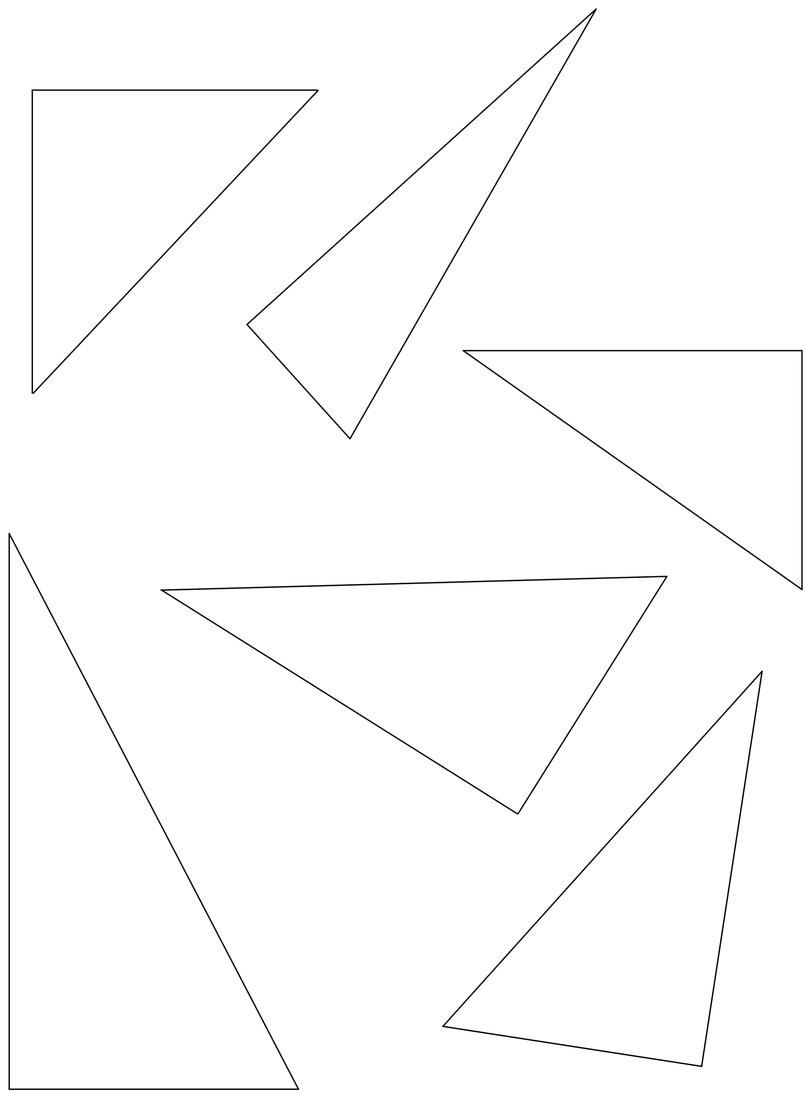


These triangles are called **equilateral triangles**.

3. (a) Measure each angle in each of the following triangles. Do you notice anything special about these angles?

....

....



(b) Identify the longest side in each of the triangles. If you are not sure which one is the longest side, measure the sides. What do you notice about the longest side in each of these triangles?

....

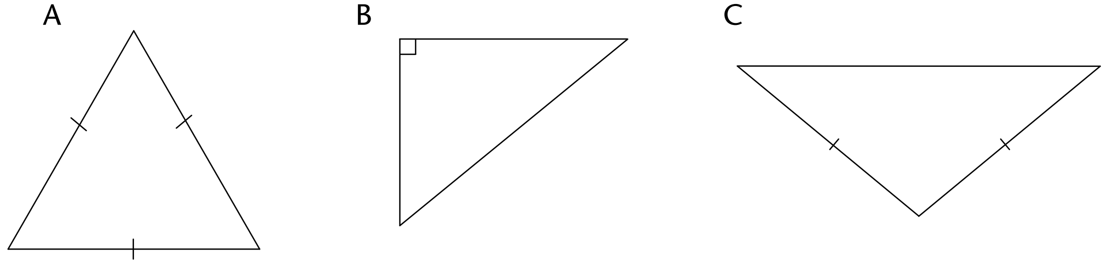
....

These triangles are called **right-angled triangles**.

COMPARING AND DESCRIBING TRIANGLES

When two or more sides of a shape are equal in length, we show this using short lines on the equal sides.

1. Use the following triangles to answer the questions that follow:



   (a) Which triangle has only two sides that are equal? ....

   What is this type of triangle called? ....

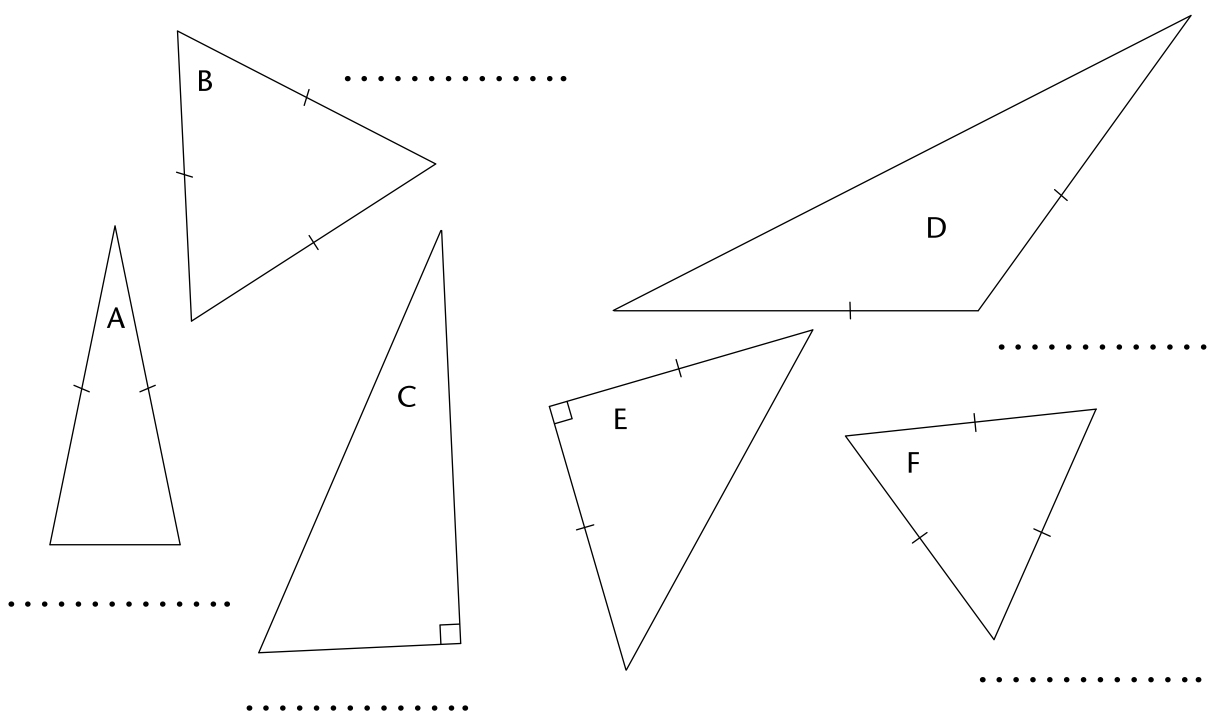
   (b) Which triangle has all three sides equal? ....

   What is this type of triangle called? ....

   (c) Which triangle has an angle equal to 90°? ....

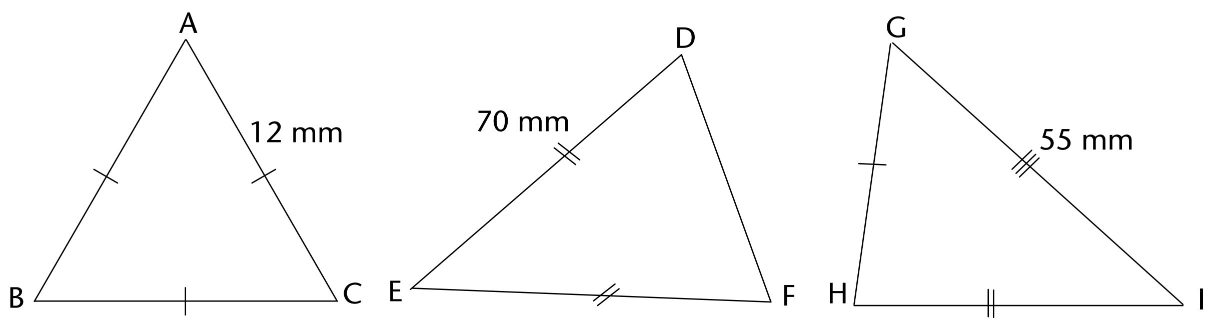
   What is this type of triangle called? ....

2. Write down the type of each of the following triangles in the space provided:



FINDING UNKNOWN SIDES IN TRIANGLES

1. (a) Name each type of triangle below.



All 3 sides are equal *2 sides are equal* *No sides are equal*

....

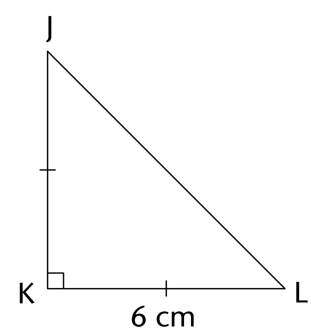
(b) Use the given information to determine the length of the following sides:

AB: .... BC: .... EF: ....

(c) Can you determine the lengths of GH and HI? Explain your answer.

....

2. The square in the corner of ΔJKL shows that it is a right angle. Give a reason for each of your answers below.



(a) Is this triangle scalene, isosceles, or equilateral?

....

(b) Name the two sides of the triangle that are equal.

....

(c) What is the length of JK?

....

(d) Name two equal angles in this triangle.

....

(e) What is the size of $\hat{J}$ and $\hat{L}$?

....

5.3 Different types of quadrilaterals

INVESTIGATING QUADRILATERALS

1. The two pages that follow show different groups of quadrilaterals.
   (a) In which groups are both pairs of opposite sides parallel? ....
   (b) In which groups are only some adjacent sides equal? ....
   (c) In which groups are all four angles equal? ....
   (d) In which groups are all the sides in each quadrilateral equal? ....
   (e) In which groups are all four sides equal? ....
   (f) In which groups is each side perpendicular to the sides adjacent to it? ....
   (g) In which groups are opposite sides equal? ....
   (h) In which groups is at least one pair of adjacent sides equal? ....
   (i) In which groups is at least one pair of opposite sides parallel? ....
   (j) In which groups are all the angles right angles? ....

2. The figures in group 1 are called **parallelograms**.
   (a) What do you observe about the opposite sides of parallelograms?

   ....
   (b) What do you observe about the angles of parallelograms?

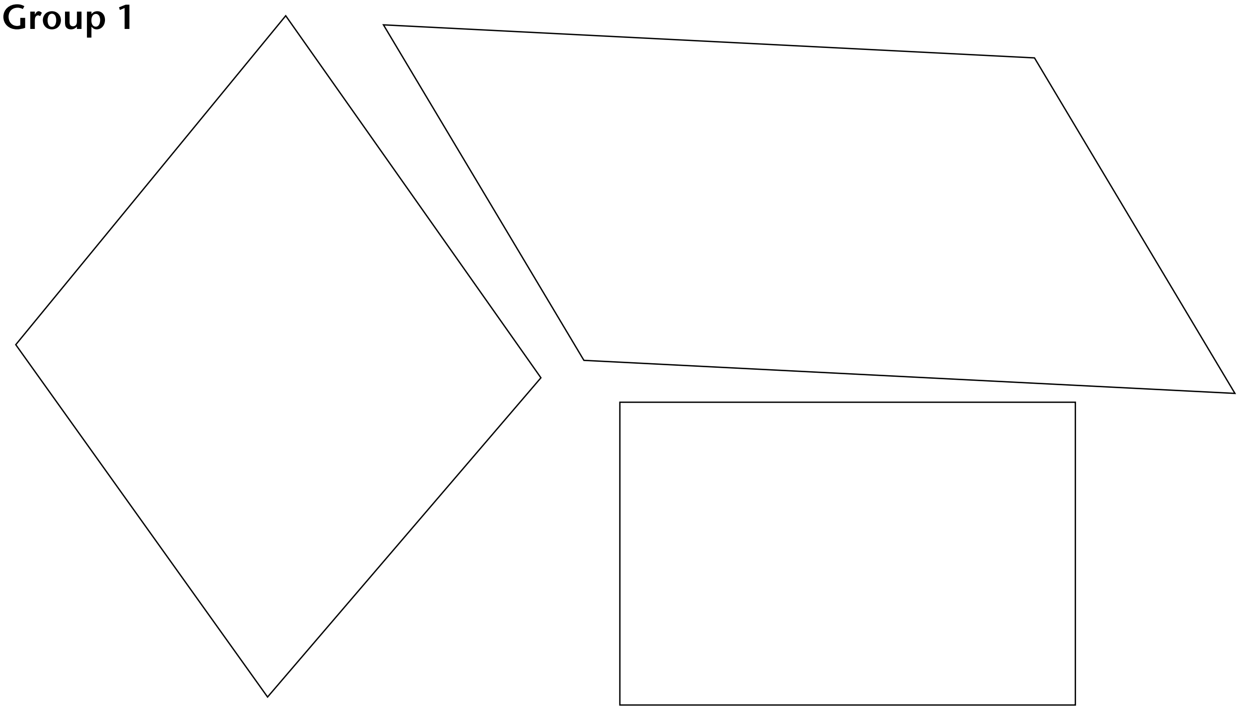
   ....

3. The figures in group 2 are called **kites**.
   (a) What do you observe about the sides of kites?

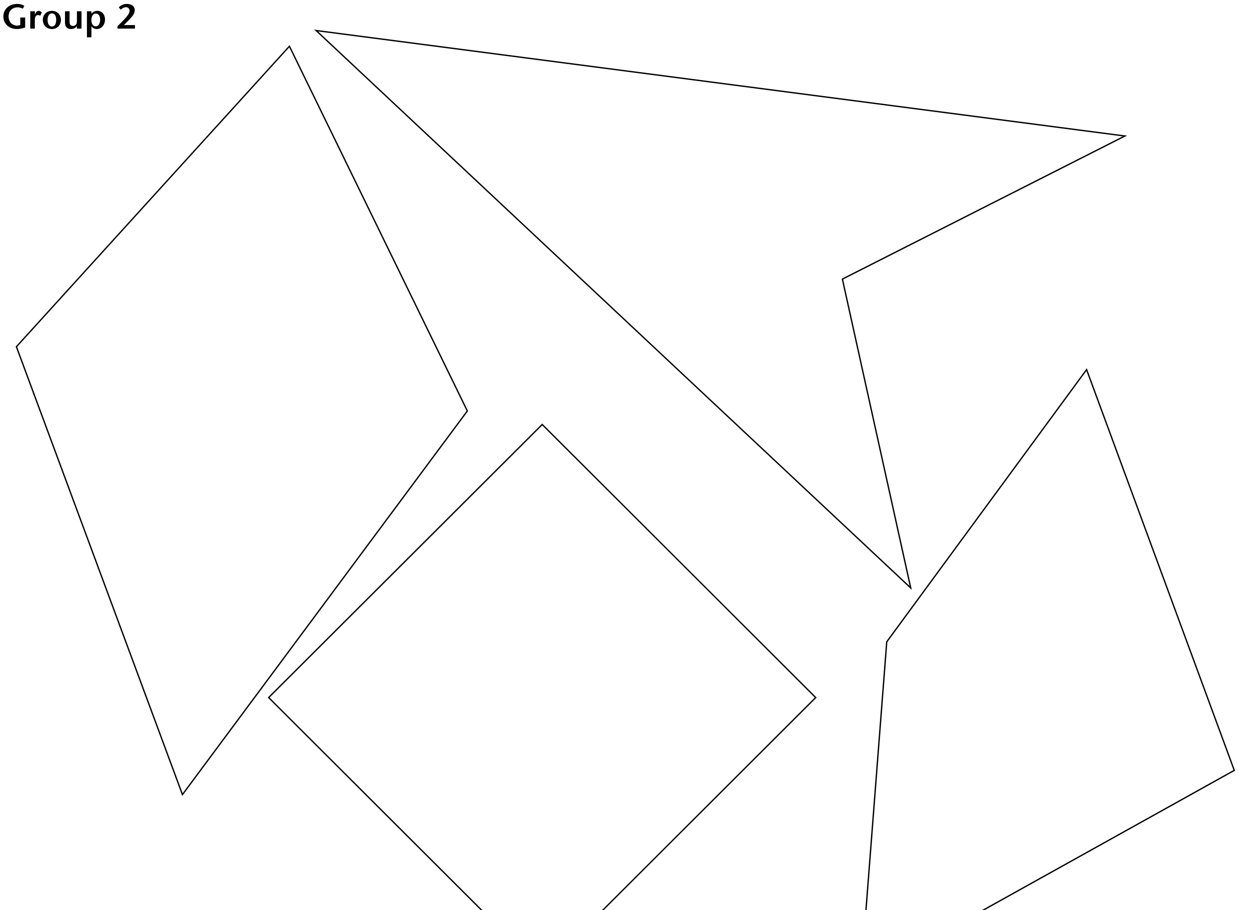
   ....
   (b) What else do you observe about the kites?

   ....

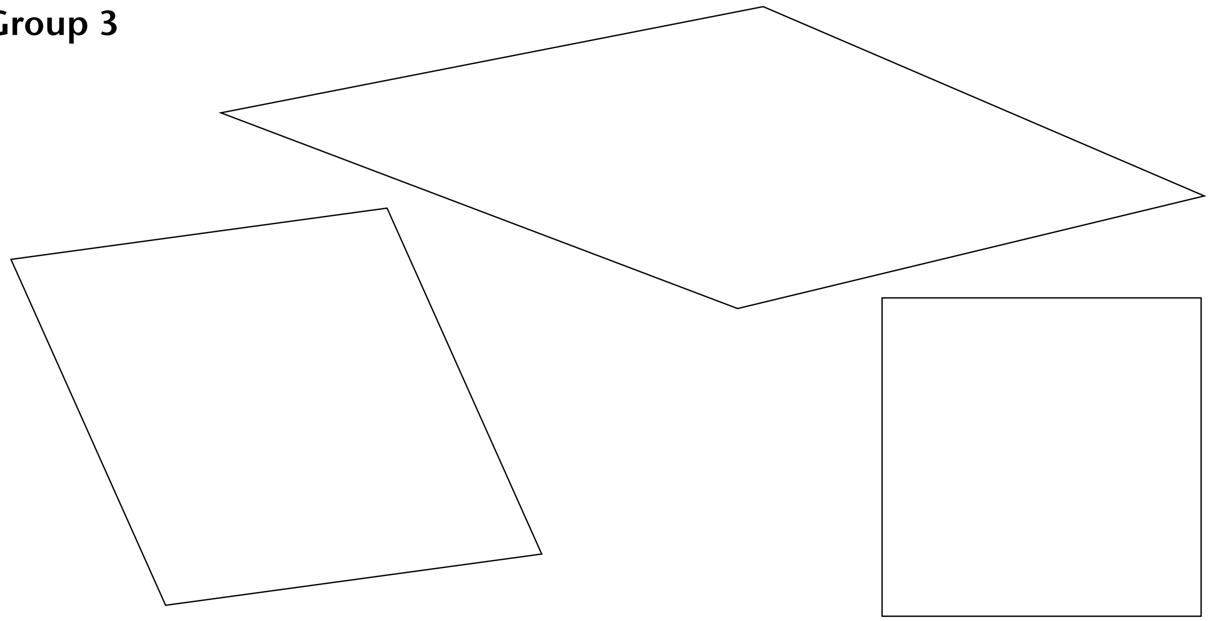
Group 1



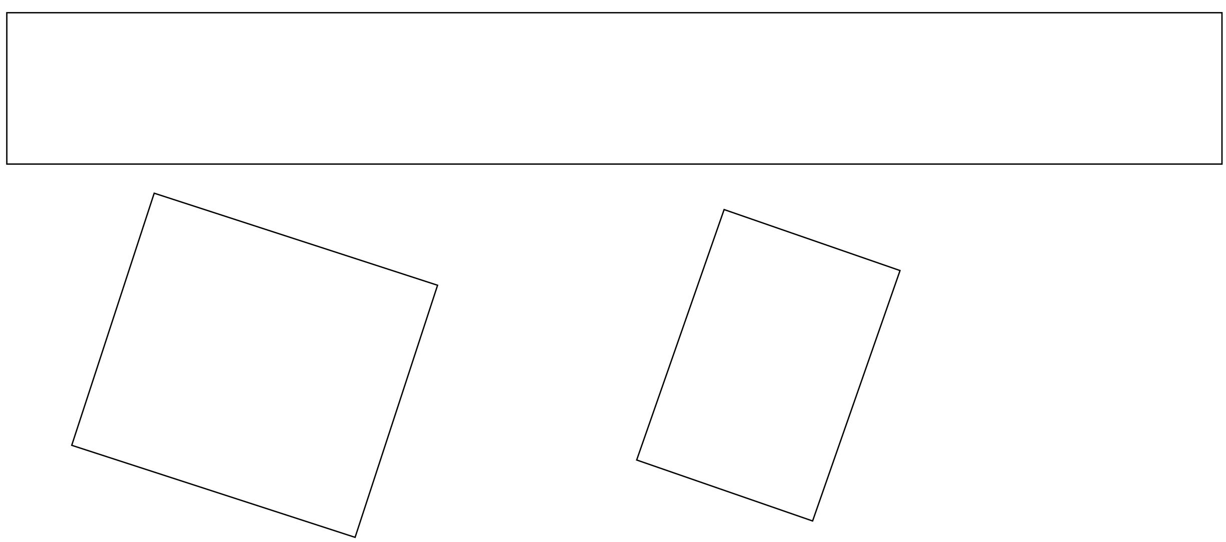
Group 2



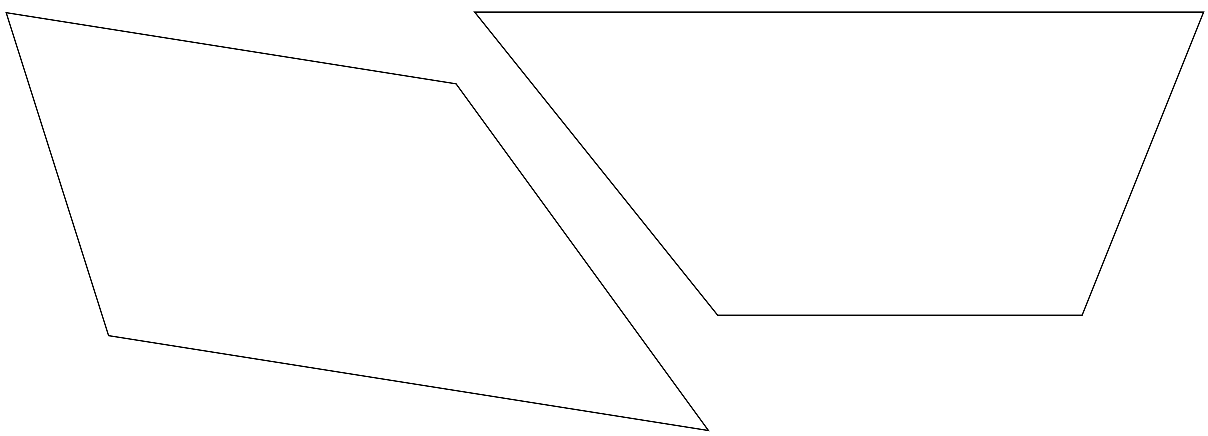
Group 3



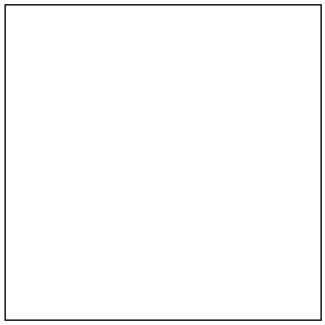
Group 4

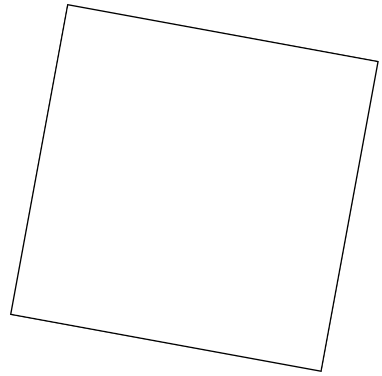


Group 5



Group 6





4. The figures in group 3 are called **rhombi**.

   (a) What do you observe about the sides of rhombi?

   ....

   (b) What else do you observe about the rhombi?

   ....

Note:
One **rhombus**; two or more **rhombi**.

5. The figures in group 4 are called **rectangles**.

   (a) What do you observe about the opposite sides of rectangles?

   ....

   (b) What do you observe about the angles of rectangles?

   ....

   (c) What do you observe about the adjacent sides of rectangles?

   ....

6. The figures in group 5 are called **trapeziums**.
   What do you observe about the opposite sides of trapeziums?

   ....

The arrows show which sides are parallel to each other.

7. The figures in group 6 are called **squares**.

   (a) What do you observe about the sides of squares?

   ....

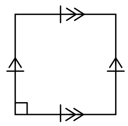
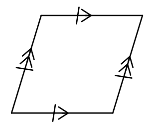
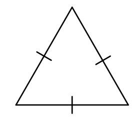
   (b) What do you observe about the angles of squares?

   ....

COMPARING AND DESCRIBING SHAPES

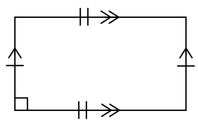
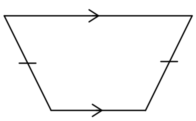
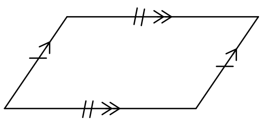
1. Name each shape in each group.

Group A

....

Group B

....

2. In what way(s) are the figures in each group the same?

Group A:

....

....

Group B:

....

....

3. In what way(s) does one of the figures in each group differ from the other two figures in the group?

Group A:

....

....

....

Group B:

....

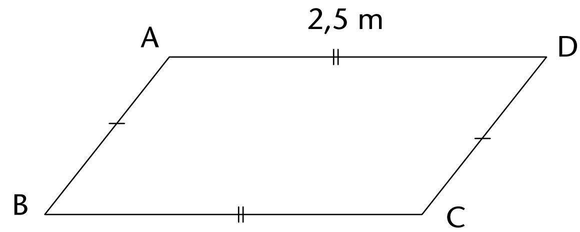
....

....

FINDING UNKNOWN SIDES IN QUADRILATERALS

Use what you know about the sides and angles of quadrilaterals to answer the following questions. **Give reasons for your answers.**

1. (a) What type of quadrilateral is ABCD?



....

....

....

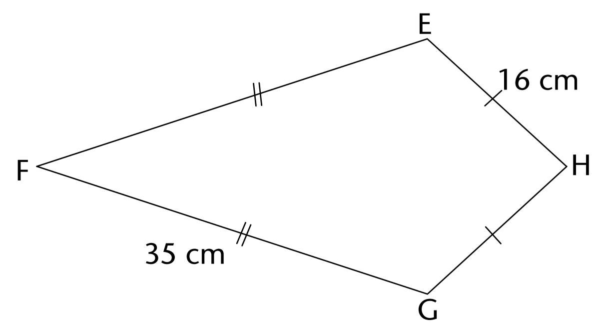
(b) Name a side equal to AB.

....

(c) What is the length of BC?

....

2. (a) What type of quadrilateral is EFGH?



....

....

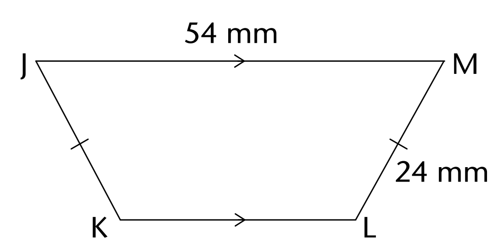
....

(b) What are the lengths of the following sides?

EF: ....

GH: ....

3. (a) What type of quadrilateral is JKLM?



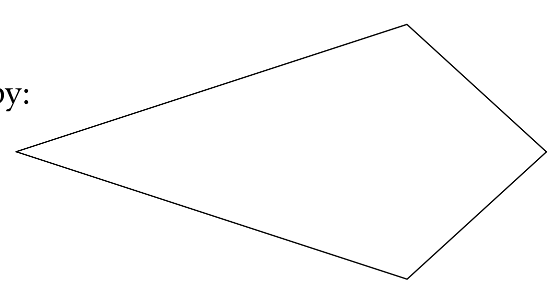
....

....

(b) What is the length of JK?

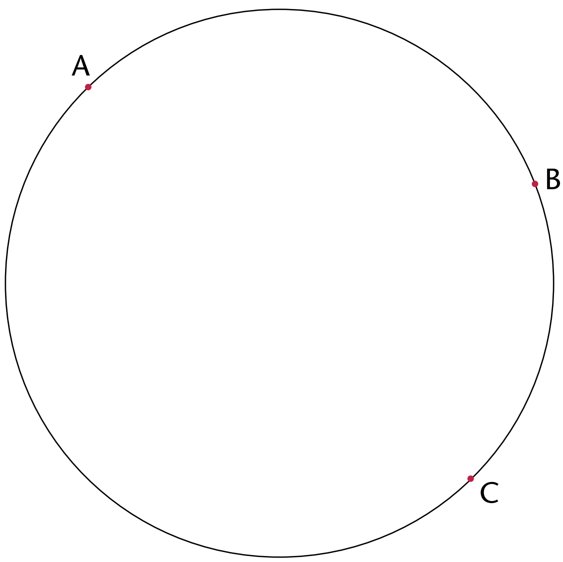
....

4. Figure PQRS is a kite with PQ = 4 cm and QR = 10 cm. Complete the following drawing by:
   (a) labelling the vertices of the kite
   (b) showing on the drawing which sides are equal
   (c) labelling the length of each side.



5.4 Circles

1. (a) Make a dot in the middle of the circle on the right. Write the letter M next to the dot. If your dot is in the middle of the circle, it is called the **midpoint** or **centre**.
   (b) Draw lines MA, MB and MC from M to the red points A, B and C.



The three red points are on the circle with midpoint M.

A straight line, such as AC, drawn across a circle and passing through its midpoint is called the **diameter** of the circle.

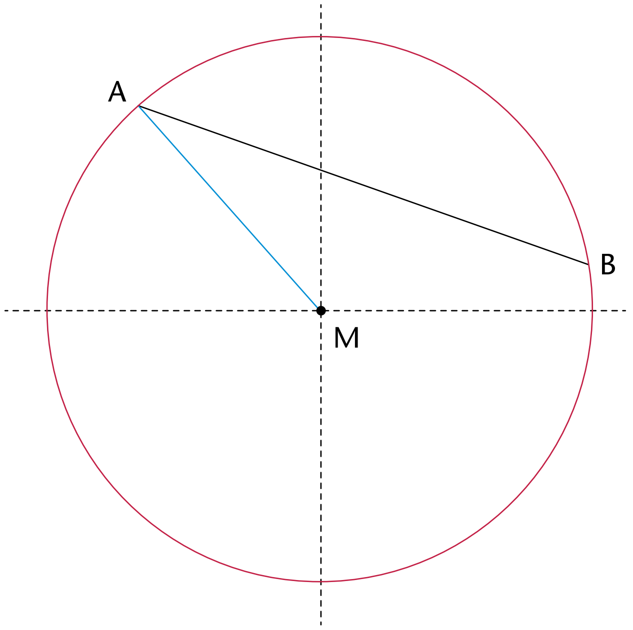
2. Measure MA, MB and MC. ....

   If MA, MB and MC are equal in length, you have chosen the midpoint well.
   If they are not equal, you may wish to improve your sketch of a circle and its parts.

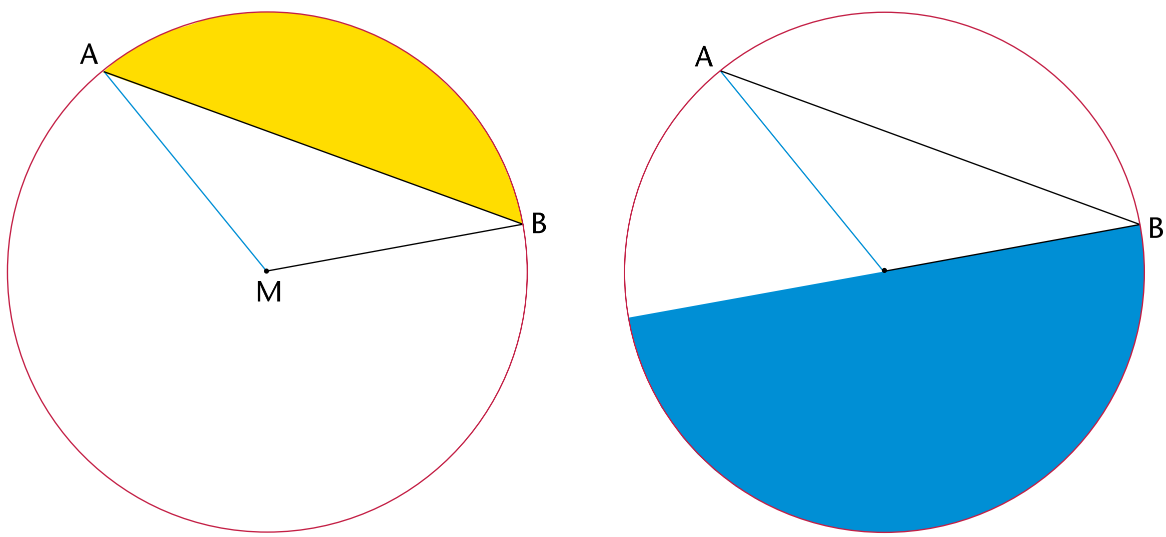
A straight line from the midpoint of a circle to a point on the circle is called a **radius** of the circle.

The blue line, MA, is a **radius**. Any straight line from the centre to the circle is a radius.

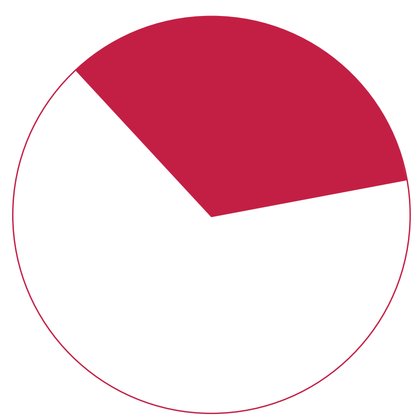
The black line AB joins two points on the circle. We call this line a **chord** of the circle.



In the following two diagrams, the coloured sections are **segments** of a circle. A segment is the area between a chord and an arc.



In the circle on the right, the red section is called a **sector** of a circle. As you can see, a sector is the region between two radii and an arc.



5.5 Similar and congruent shapes

Three groups of quadrilaterals are shown on this page and the next.

What makes each group different from the other groups, apart from the colours?

1. Group A:

   ....

   ....

2. Group B:

   ....

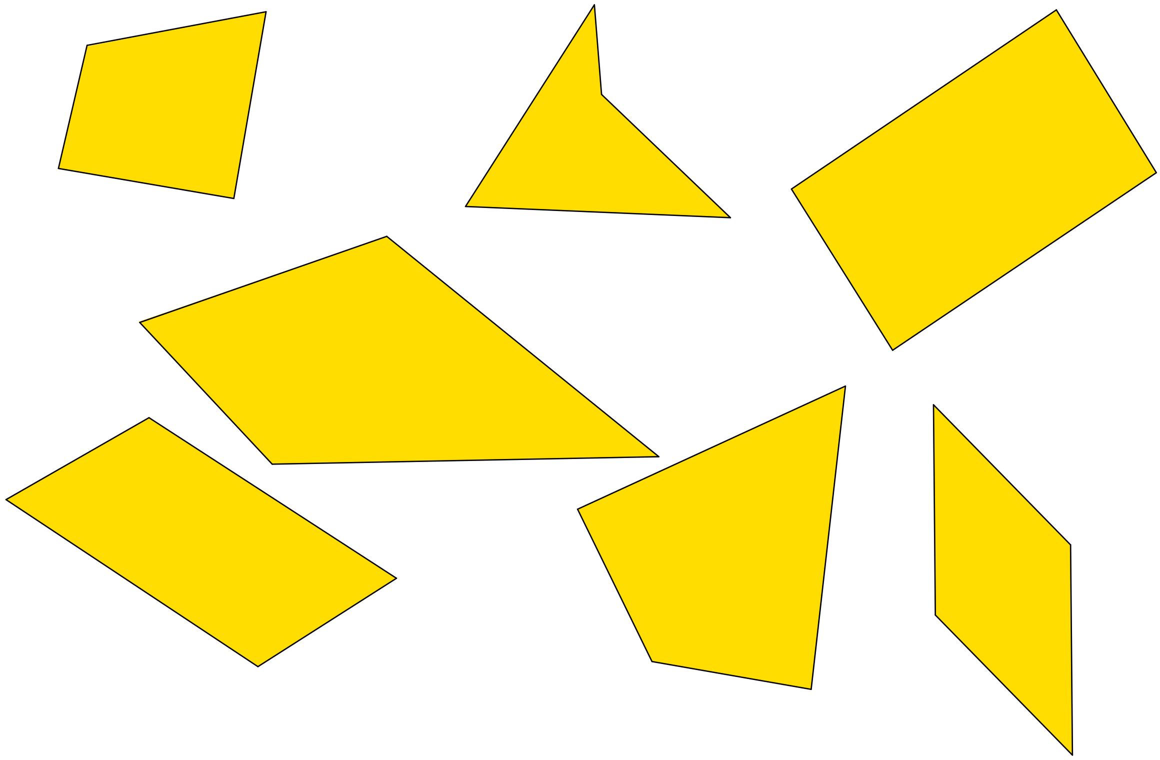
   ....

3. Group C:

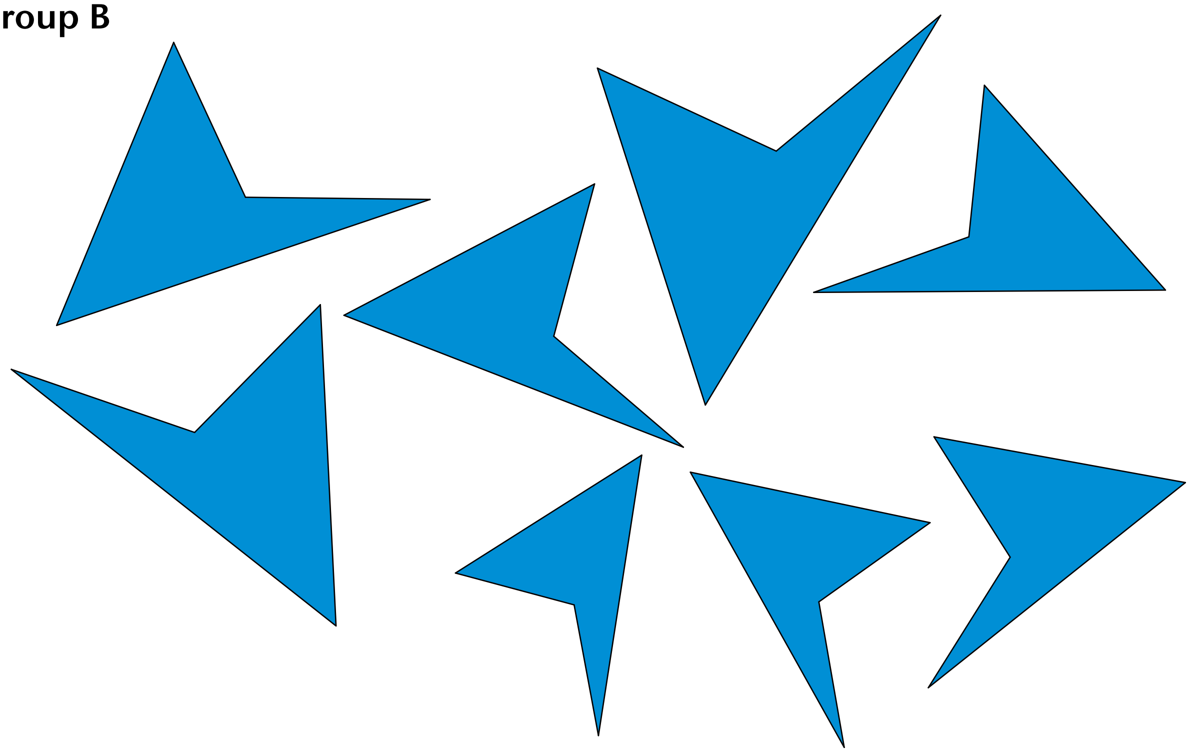
   ....

   ....

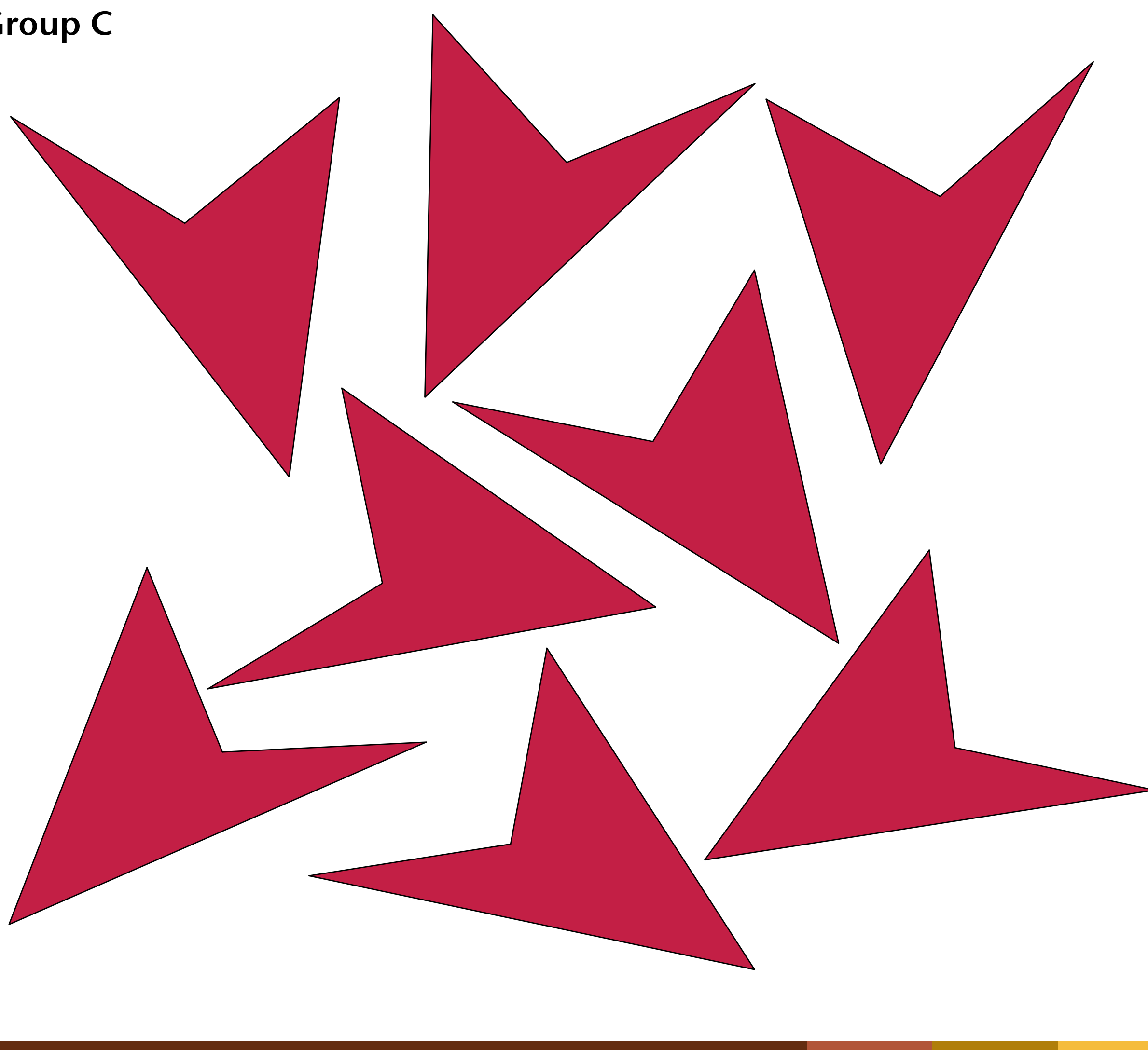
Group A



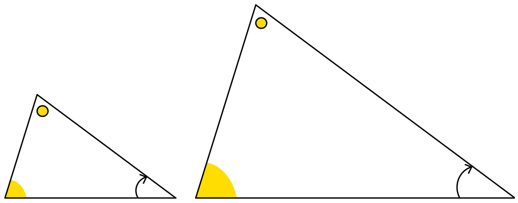
Group B



Group C

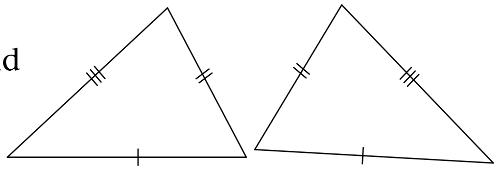


Shapes that have the same form, such as the blue shapes on the previous page, are said to be **similar** to each other. Similar shapes may differ in size, but will always have the same shape.



Example of similar shapes

Shapes that have the same form and the same size, such as the red shapes on the previous page, are said to be **congruent** to each other. These shapes are always the same size and shape.

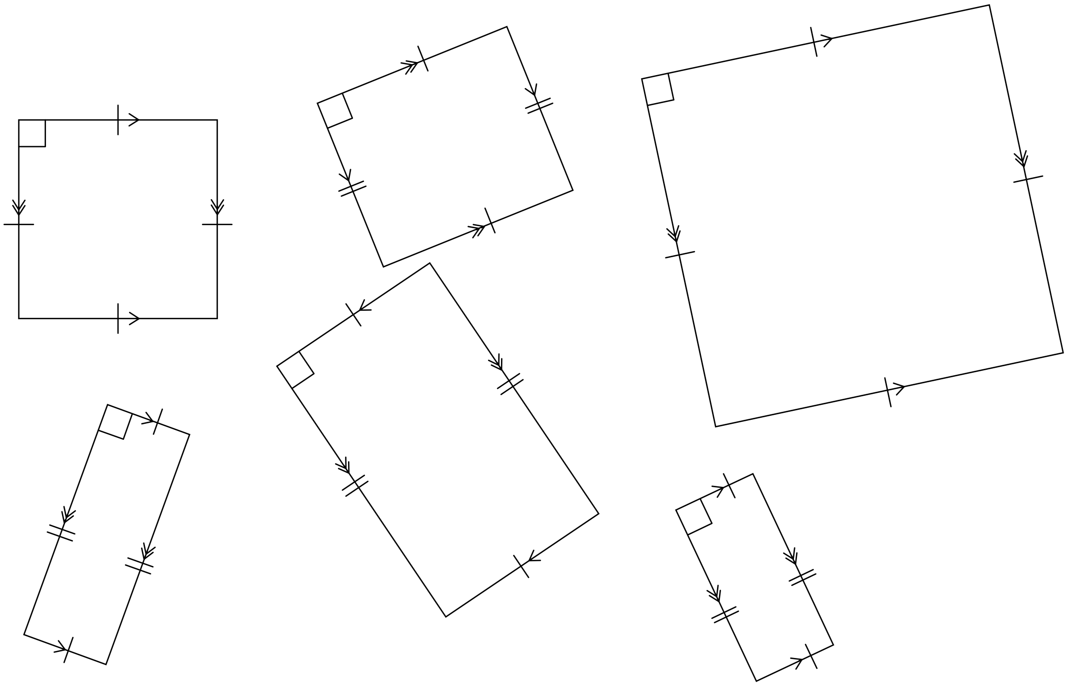


Example of congruent shapes

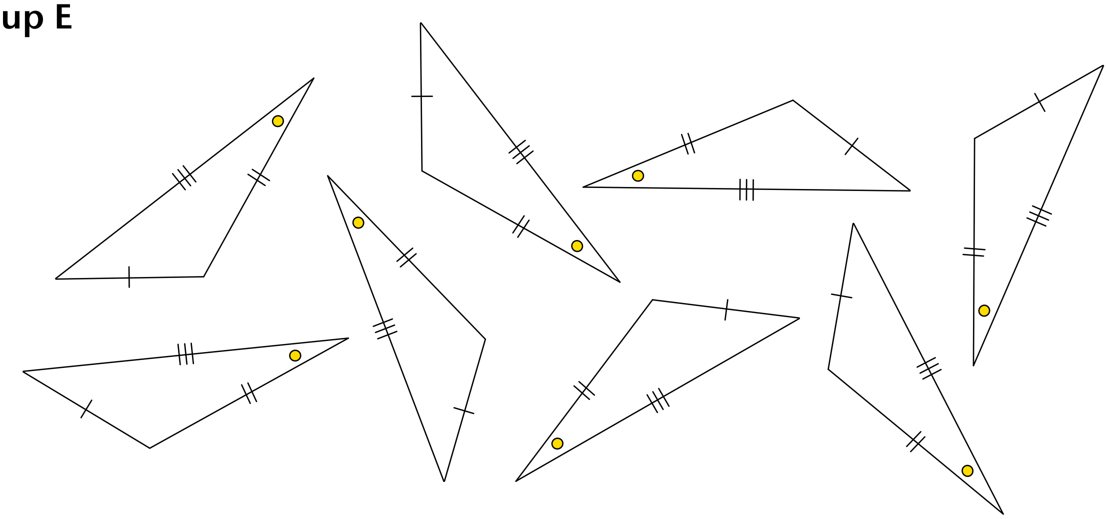
4. Are the red shapes on the previous page *similar* to each other? ....

5. Look at groups D, E, F, and G on this page and the next. In each case say whether the shapes are similar and congruent, similar but not congruent, or neither similar nor congruent.
   (a) Group D: ....
   (b) Group E: ....
   (c) Group F: ....
   (d) Group G: ....

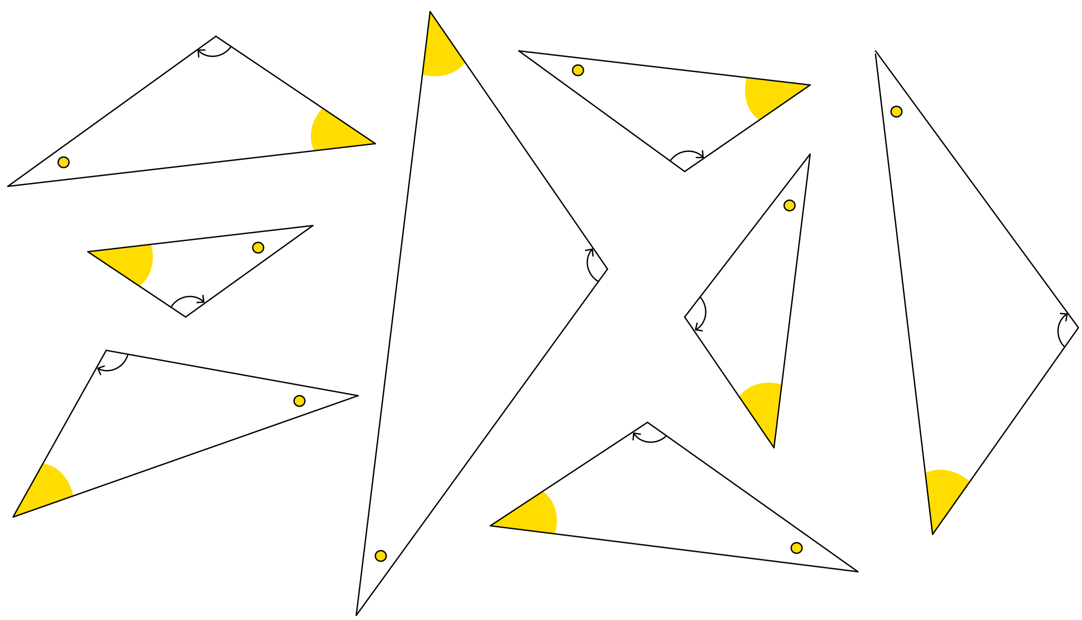
Group D



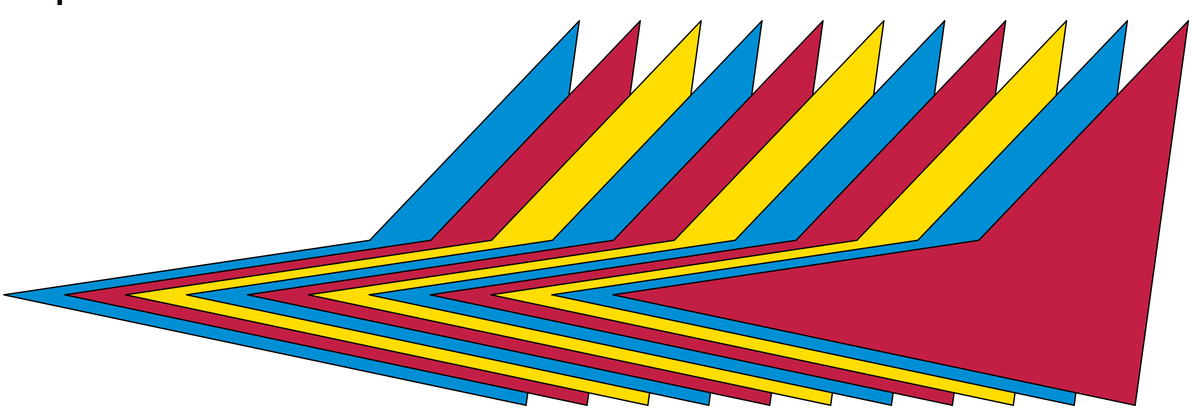
Group E



Group F



Group G



Term 1
Revision and assessment

Revision

You should not use a calculator for any of the questions in this section. Do show your steps of working.

WORKING WITH WHOLE NUMBERS

1. Make the number sentences below true. Into the first block in each question, write a multiplication or division sign. Into the second block, write either 10, 100 or 1 000.

   (a) 8 $\square$ $\square$ = 800

   (b) 740 000 $\square$ $\square$ = 740

2. Circle all the numbers given below that will round off to 60 000.

   62 495   54 498   65 000   56 002   67 024

3. Calculate the following:

   (a) 274 561 + 367 238

   (b) 4 672 – 3 937

   (c) 3 458 × 43

   (d) 6 624 ÷ 18

4. Write the missing numbers in the blocks.

   (a) 8; 15; 22; 29; $\square$

   (b) 71 590 $\square$ 71 600

5. Tumi makes a sequence of numbers using the following rule:

   "Take half the previous number and then add 12."

   Write the next three numbers in the sequence:

   56; 40; 32; ....

6. Two three-digit numbers are added together as shown, and produce a three-digit answer – but some of the digits are missing. Fill in the missing digits so as to make the calculation correct.

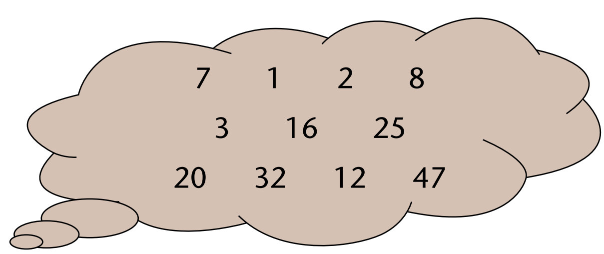
   $5\,9\,\boxed{} + 3\,\boxed{}\,9 = \boxed{}\,5\,3$

7. Ismail has the following numbers:

   71; 72; 73; 74; 75; 76; 77; 78; 79; 80

   He wishes to sort them by placing them in the sorting diagram below. Help Ismail by placing the numbers in the correct blocks.

	Prime number	**Not a prime number**
Multiple of 4		
Not a multiple of 4		



8. Write down, using *only* numbers from the cloud:

   (a) All the prime numbers

   ....

   (b) All the square numbers

   ....

   (c) All the cube numbers

   ....

   (d) All the multiples of 8

   ....

   (e) All the factors of 8

   ....

9. Teacher Ramushwana states:
"Every **even number** (greater than or equal to 6) can be written as the sum of a **pair of odd prime numbers**, for example 10 = 3 + 7."
   (a) Write down *two* pairs of odd prime numbers that each sum to 20.

   ....

   (b) Choose any even number greater than 30 and write it as a sum of two odd prime numbers.

   ....

10. (a) Write the following as a product of prime factors:
   (i) 576

   ....

   (ii) 600

   ....

   (b) Find both the (i) HCF and (ii) LCM of 576 and 600.
   (i) HCF

   ....

   (ii) LCM (leave your answer as a product of prime factors)

   ....

11. How many hours will it take the Adams family to reach their holiday destination if it is 495 km away and they travel at an average speed of 110 km/h?

....

....

....

12. Graeme, Thuli and Andile have worked as a team over the holidays, mowing the lawns of their neighbours. They collected a total of R1 200 and now need to share it. They agree that as they didn't all work an equal amount, the money should be shared between Graeme, Thuli and Andile in the ratio 4 : 6 : 5. How much money will Thuli receive?

....

....

....

13. Mr Khumalo decides to try to make some money buying and selling used furniture. He has R6 000 in his bank account, and uses some of the money to buy an old bed base and mattress for R800, a dresser for R2 500, two lockers for R300 each, and a washing machine for R900.

(a) How much is left in his bank account after these purchases?

....

....

....

....

(b) Suppose that he sells the bed base and mattress for R980, the dresser for R2 950, and both lockers for a total of R750. Nobody seems to want his washing machine though. At this stage he has made a loss. What is the value of the loss?

....

....

....

....

(c) How much does he need to sell the washing machine for to have an overall profit of R1 000?

....

....

....

14. Mrs Steyn takes out a loan of R55 000 from Fidelity Bank. The bank charges simple interest of R500 per month. How much money will Mrs Steyn owe after $1\frac{1}{2}$ years?

....

....

....

15. John earns R480 on a Saturday. He works from 08:00 to 14:00. Calculate his hourly rate.

....

EXPONENTS

1. Calculate.
   (a) 12×12 .... (b) 8×8 ....
   (c) $7 \times 7 \times 7$ .... (d) $3 \times 3 \times 3$ ....
   (e) $6 \times 6 \times 6$ .... (f) 13×13 ....

2. Explain the difference between 4×3 and 4^3.

   ....

3. Write 5^5 in expanded form.

   ....

4. Write the following in exponential form:
   (a) $2 \times 2 \times 2$ .... (b) $3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$ ....

5. Write the numbers in exponential form. Check your answers.
   (a) 81 ....
   (b) 10 000 ....

6. Complete: (a) 5 squared is ....
   (b) 5 cubed is ....

7. Calculate:
   (a) 7^2 (b) 15^2

   ....

   (c) $3^2 \times 4^2$ (d) $\sqrt{16}$

   ....

8. Are the following statements true or false? Explain your answers.
   (a) The number 64 can be written both as a square and a cube.

   ....

   ....

   (b) 2^5 is smaller than 30^1.

   ....

   ....

9. Calculate the following and give reasons for your answers.

(a) $\sqrt[3]{216}$

(b) $\sqrt[3]{8}$

(c) $\sqrt[3]{125}$

(d) $\sqrt[3]{27}$

10. Determine the value of each of the following:

(a) $3^2 - 2^3$

(b) $4(10 - 1^{100})$

(c) $(8 - 2)^2$

(d) $\sqrt{4} \times \sqrt{81}$

(e) $(\sqrt{58})^2$

(f) $\sqrt[3]{27} \div \sqrt{9}$

(g) $10 \times \sqrt{81}$

(h) $\sqrt[3]{2 \times 32}$

11. $13^2 = 169$; $14^2 = 196$; $15^2 = 225$; $16^2 = 256$; $7^3 = 343$; $8^3 = 512$; $3^3 = 27$

Use these facts to calculate the value of each of the following:

(a) $\sqrt{196} - \sqrt[3]{512}$

(b) $\sqrt{169 \times 225}$

(c) $\dfrac{\sqrt{196}}{\sqrt[3]{343}}$

(d) $\dfrac{14^2 - 13^2}{3^3}$

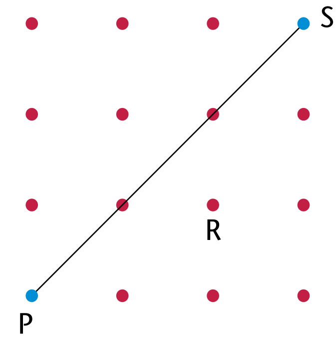
12. If $56^3 = 175\,616$, write down the value of $\sqrt[3]{175\,616}$.

GEOMETRY OF STRAIGHT LINES

1. Consider the grid shown alongside.
   (a) Is PS a line, ray or line segment?

   ....

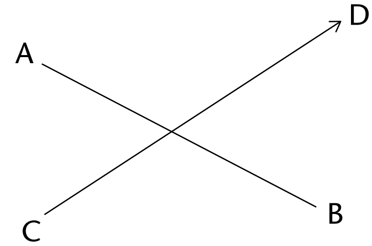
   (b) Draw on the grid a line segment through R that will be perpendicular to PS. Label it TU.
   (c) Draw on the grid a line that is parallel to PS. Label it WX.



2. Provide the correct name for each of the geometric features AB and CD, shown on the diagram:

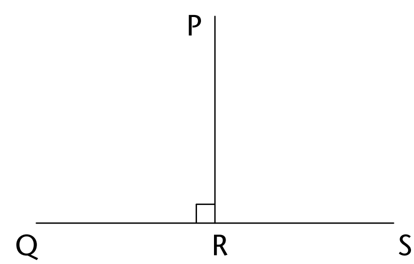
   AB: ....

   CD: ....



3. There is a geometric relationship between line segments PR and QS shown in the diagram. Describe the relationship by adding the correct word on the dotted line:

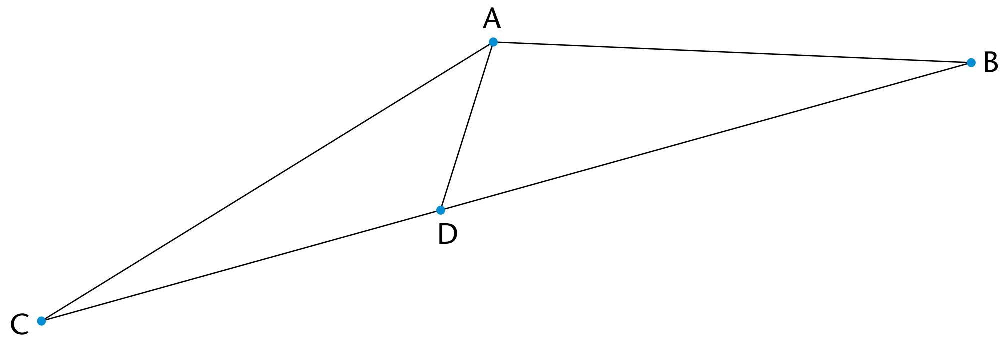
   PR is .... to QS.



4. Draw a ray and a line that will never meet.

CONSTRUCTION OF GEOMETRIC FIGURES

1. Use a protractor to accurately measure the following angles, as shown on the diagram below, and write the answers in the table provided:
   (a) $\hat{B}$
   (b) $A\hat{D}B$
   (c) $D\hat{A}B$
   (d) $C\hat{D}B$
   (e) reflex $C\hat{A}B$



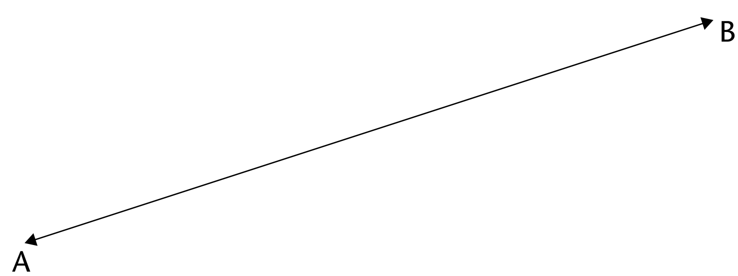
Angle name	Size	Classification
$\hat{B}$		
$A\hat{D}B$		
$D\hat{A}B$		
$C\hat{D}B$		
Reflex $C\hat{A}B$		Reflex

2. Construct a semi-circle with a radius of 3 cm.

3. Use a ruler and protractor to construct angles that are the given sizes. Label the angles correctly.

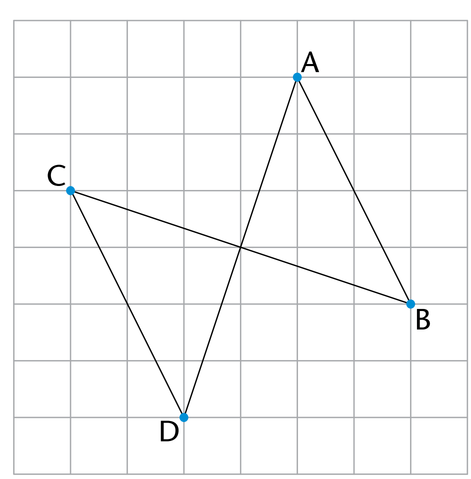
   (a) $E\hat{F}G = 152°$ (b) $X\hat{Y}Z = 289°$

4. Use circles to construct two lines, CD and EF, that are parallel to line AB below. Line CD should be above line AB, and line EF below line AB. Label both lines.



5. Consider the diagram alongside. Write down the names of the pair of perpendicular lines.

   ....



GEOMETRY OF 2D SHAPES

1. Give the full name of the shape that fits the given descriptions:
   (a) A three-sided shape that has exactly two sides equal in length

   ....

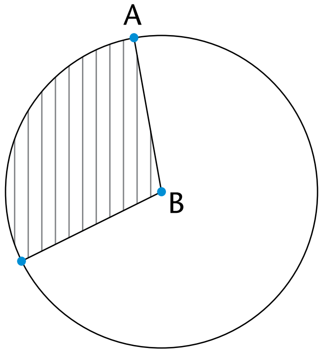
   (b) A four-sided shape with both pairs of opposite sides parallel and equal in length, and with no right angles

   ....

   (c) A four-sided shape with only one pair of opposite sides parallel

   ....

2. What is the correct term for each of the following parts of the circle with centre B, as shown alongside?



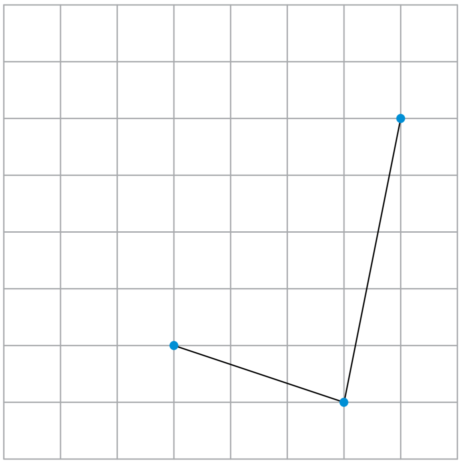
   (a) Line AB

   ....

   (b) The shaded area

   ....

3. On the square grid alongside, two sides of a kite have been drawn. Use a ruler and complete the kite on the grid.



4. Darrel says, "The four-sided shapes I am thinking of have at least one pair of adjacent sides equal. What are they?" Write down the names of all the shapes that fit his description.

   ....

   ....

   ....

5. DEFG is a kite, and DE = 4 cm and EF = 5,2 cm. Write down the lengths of DG and GF.

   ....

   ....

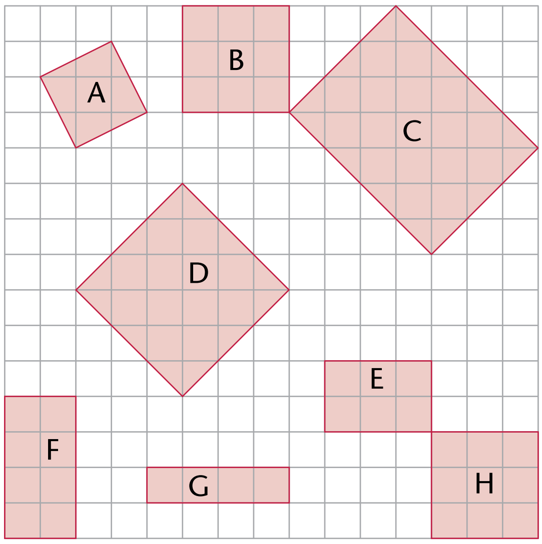
6. STUV is a rectangle. Write down the value of $\hat{T} + \hat{V}$.
   Give a reason for your answer.

   ....

   ....

   ....

7. Consider the diagram alongside.



   (a) Write down the letters of all the shape(s) that are *congruent* to shape B.

   ....

   (b) Write down the letters of all the shape(s) that are *similar* to shape B.

   ....

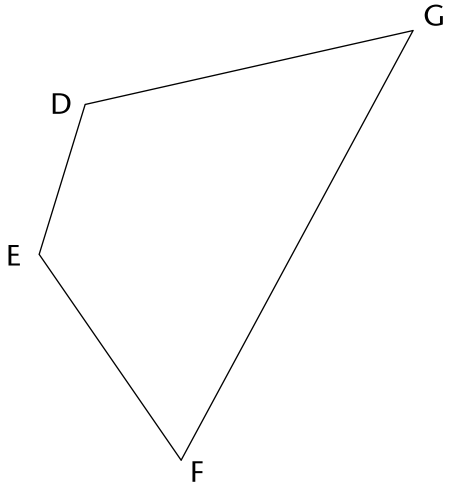
8. An isosceles triangle, LMN, has LM = 4 cm and a perimeter of 16 cm. Investigate and write down *all* the possible lengths of MN and LN.

   ....

   ....

   ....

9. In each case say whether the two sides are opposite sides or adjacent sides of quadrilateral DEFG.



   (a) GD and DE

   ....

   (b) DE and GF

   ....

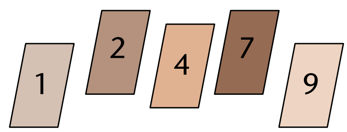
Assessment

In this section, the numbers indicated in brackets at the end of a question indicate the number of marks that this question is worth. Use this information to help you determine how much working is needed.

The total number of marks allocated to the assessment is 60.

Note:
Do not use your calculator!

1. Here are five one-digit cards:



Two-digit numbers can be made by placing two cards next to each other – so, for example, taking the 1 and the 2 and putting them next to each other will create the number 12 (twelve).

Choose two cards each time to make the following two-digit numbers: (4)

(a) An odd number

....

(b) A multiple of 9

....

(c) A factor of 126

....

(d) A square number

....

2. Ayanda has a pack of cards numbered from 1 to 16. He chooses four cards at random from the pack:

- One is a factor of 39.
- Two are multiples of 4.
- Three are even.
- The total of the four numbers is more than 45, but less than 50.

Write down the values of the four numbers. (3)

....

3. All 769 learners from the Sibanye Primary School are going to an athletics meeting. The school hires buses from a local company. Each bus can take only 52 passengers, and each bus has to have two teachers on board.

   How many buses will the school have to hire to get everyone to the meeting? (3)

   ....

   ....

   ....

4. Calculate the total number of test wickets taken by the top four wicket takers in the Proteas' cricket history (figures correct as at June 2013):
   - Shaun Pollock: 421
   - Makhaya Ntini: 390
   - Dale Steyn: 332
   - Allan Donald: 330 (2)

   ....

   ....

   ....

   ....

   ....

5. Use prime factors to find the LCM of 42 and 18. (2)

   ....

   ....

   ....

6. Dintle's family needs to get to Polokwane, 330 km away, by 11 a.m. If they leave at 7.40 a.m. and drive at an average speed of 100 km/h, will they reach their destination on time? Show all your working. (3)

   ....

   ....

   ....

   ....

7. Determine the value of each of the following:

(a) $6^3 - (7^2 + 6^2)$ (3)

....

....

....

(b) $(8 - 5)^3$ (2)

....

....

(c) $(\sqrt[3]{125})^2$ (2)

....

....

(d) $12^2 - 4\sqrt{121} \div 2^2$ (4)

....

....

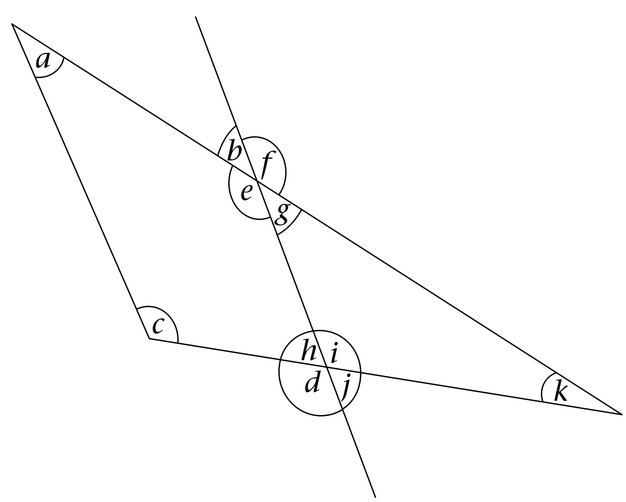
(e) $3\sqrt{64}$ (2)

....

....

8. (a) Write down the letters of all the acute angles in the diagram. (2)

....

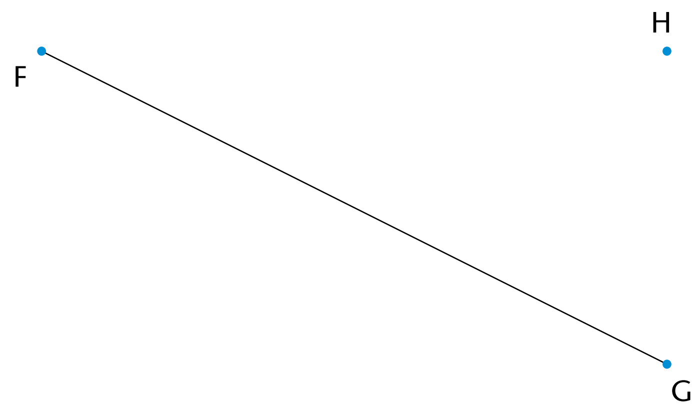


(b) Measure the size of angle d in the diagram and write it down. .... (2)

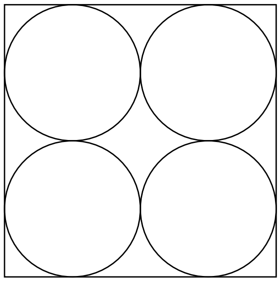
(c) Classify angle d according to its size. .... (1)

9. Draw and correctly label angle KLM = 168°. (3)

10. Use your ruler and protractor to draw a line that is parallel to line segment FG drawn below, and goes through point H. (3)



11. Four circles are drawn so that they fit neatly into a square with side length of 6 cm, as shown (*not* to scale). Write down the radius of each circle. (2)



....

....

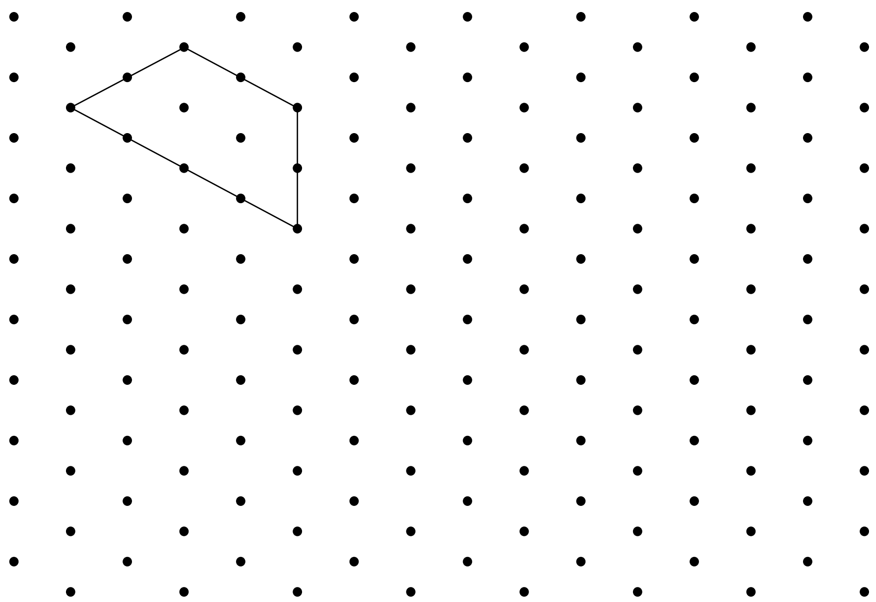
....

....

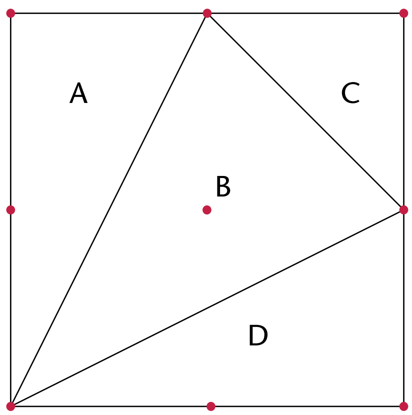
12. (a) What is the geometrical name of the shape shown on the dot grid below? (1)

. . . .

(b) Draw two shapes that are *similar* to the shape shown, anywhere on the grid. Each shape that you draw should have a different size. (4)



13. The following diagram shows a square drawn on a dot grid. The square is divided into four triangles, namely A, B, C, and D.



(a) Write down the letters of all the right-angled triangles. (1)

. . . .

(b) Write down the letters of all the isosceles triangles. (1)

. . . .

(c) Write down the letters of the two congruent triangles. (1)

. . . .

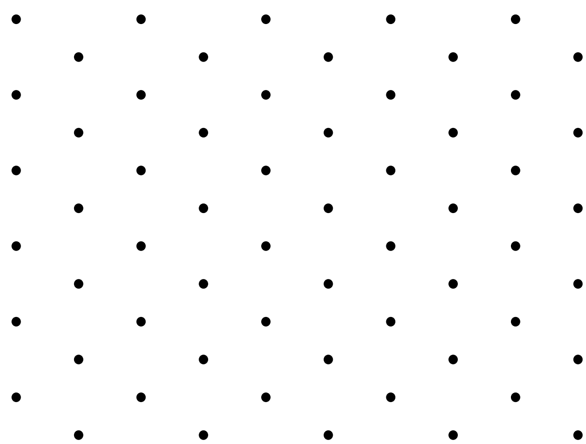
14. I am a quadrilateral with two pairs of opposite sides equal, no adjacent sides equal, and no right angles. What shape am I? (2)

. . . .

15. What is the special name we give to the perimeter of a circle? (1)

....

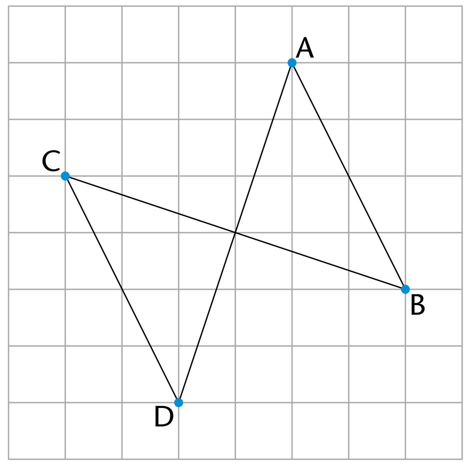
16. Draw a rhombus of any size on the dot grid below. Add appropriate symbols on the diagram to show that the opposite sides of a rhombus are parallel. (3)



17. In trapezium JKLM, JK is parallel to the opposite side.
Complete the statement:

JK || ..... (1)

18. Study the following diagram:



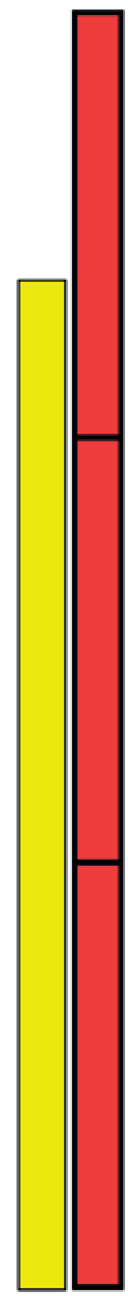
Cross out the *incorrect* word or symbol in each set of brackets:

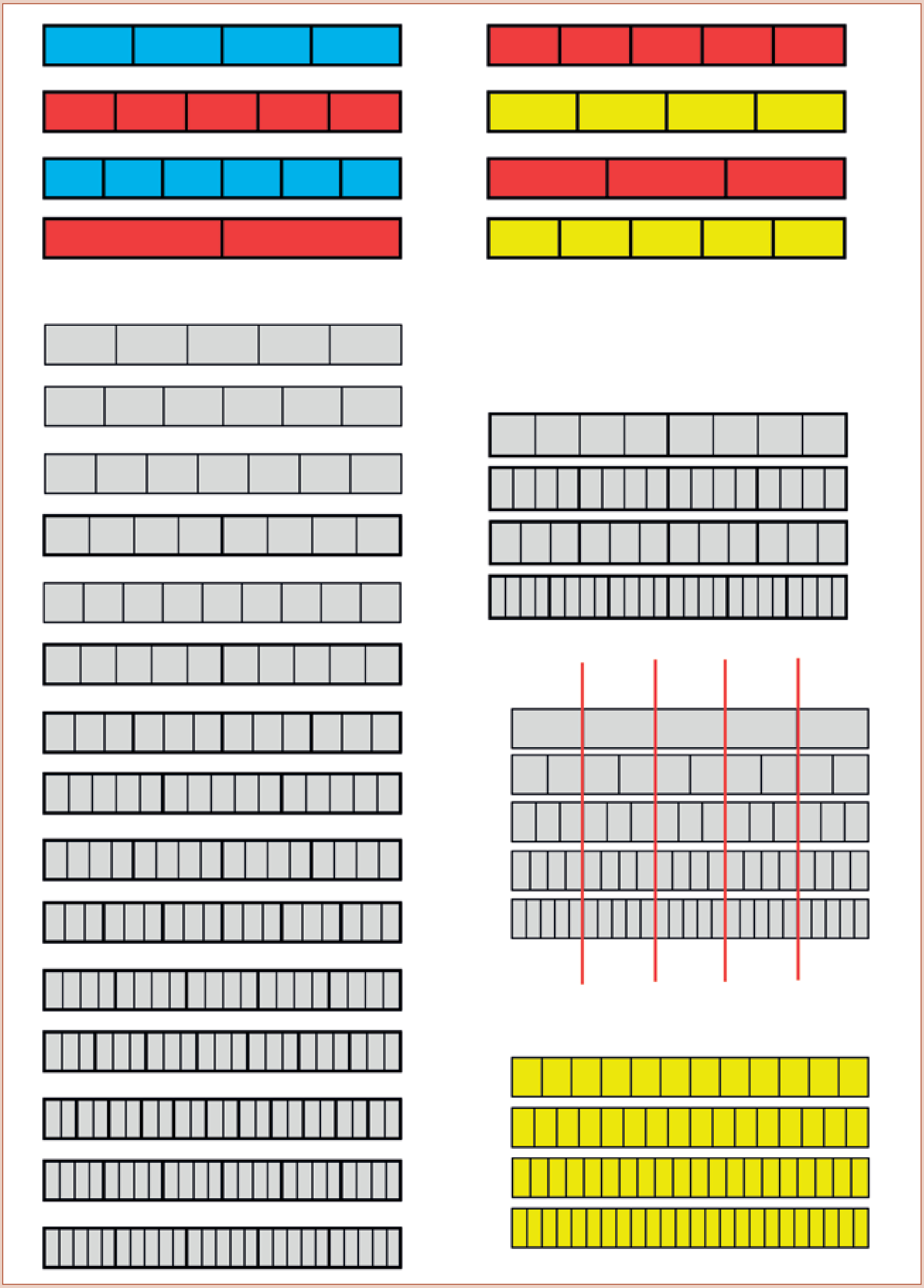
AD is (parallel/perpendicular) to BC. This can be shown in symbols as follows:

(AD ⊥ BC / AB || BC). (2)

Chapter 6
Fractions

In this chapter you will learn how to say precisely how long something is. With whole numbers only, we cannot always say precisely how long something is. Fractions were invented for that purpose. You will also learn to calculate with fractions.





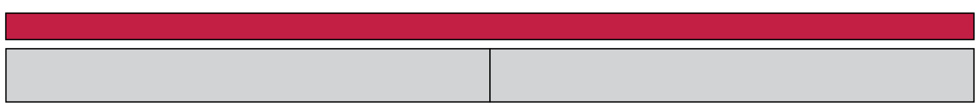
6 Fractions

6.1 Measuring accurately with parts of a unit

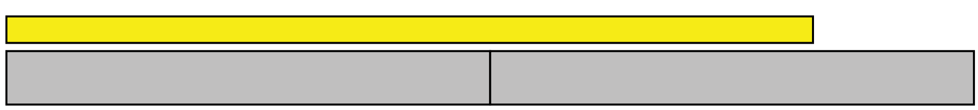
A STRANGE MEASURING UNIT

In this activity, you will measure lengths with a unit called a *greystick*. The grey measuring stick below is exactly 1 greystick long. You will use this stick to measure different objects.

The red bar below is exactly 2 greysticks long.



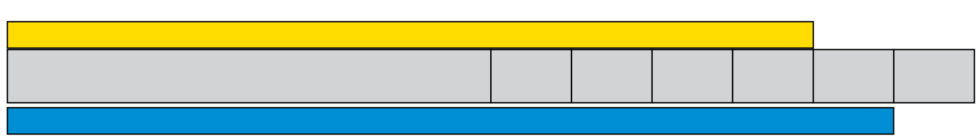
As you can see, the yellow bar below is longer than 1 greystick but shorter than 2 greysticks.



To try to measure the yellow bar accurately, we will divide one greystick into six equal parts:

So each of these parts is **one sixth** of a greystick.

1. Do you think one can say the yellow bar is **one and four sixths of a greystick** long? ....



2. Describe the length of the blue bar in words.

....

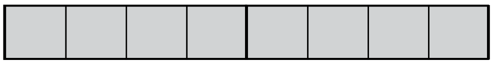
This greystick ruler is divided into seven equal parts:

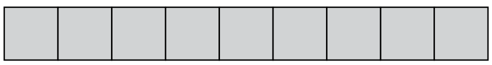
Each part is **one seventh** of a greystick.

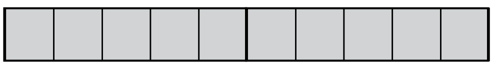
3. In each case below, say what the smaller parts of the greystick may be called. Write your answers in words.

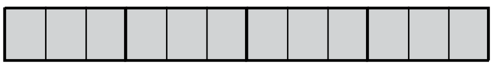
(a)  ....

(b)  ....

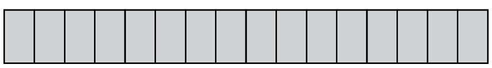
(c)  ....

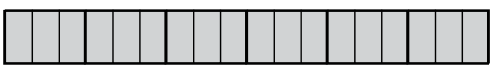
(d)  ....

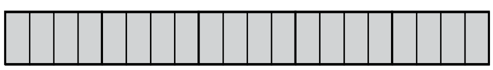
(e)  ....

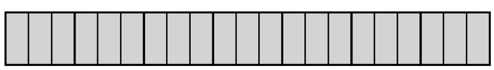
(f)  ....

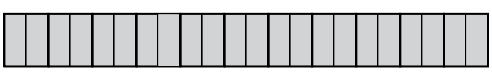
(g)  ....

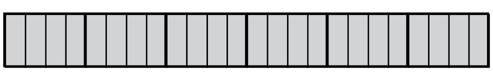
(h)  ....

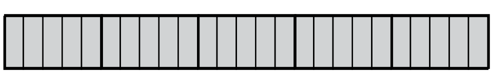
(i)  ....

(j)  ....

(k)  ....

(l)  ....

(m)  ....

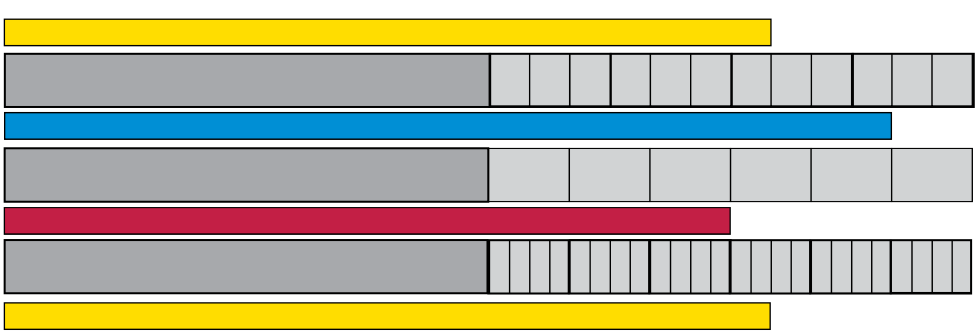
(n)  ....

How did you find out what to call the small parts? ....

....

Write all your answers to the following questions *in words*.

4. (a) How long is the upper yellow bar? ....



(b) How long is the lower yellow bar? ....

5. (a) How long is the blue bar at the bottom of the previous page?

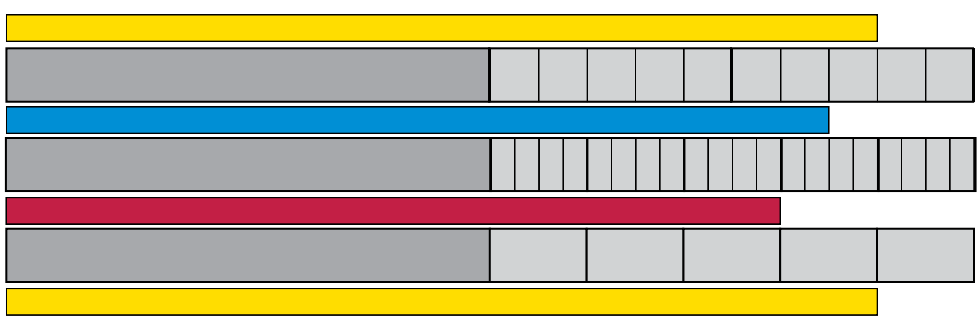
(b) How long is the red bar at the bottom of the previous page?

6. (a) How many twelfths of a greystick is the same length as one sixth of a greystick?

(b) How many twenty-fourths is the same length as one sixth of a greystick?

(c) How many twenty-fourths is the same length as seven twelfths of a greystick?

7. (a) How long is the upper yellow bar below?

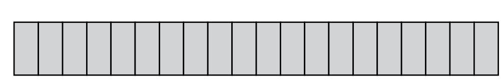


(b) How long is the lower yellow bar above?

(c) How long is the blue bar?

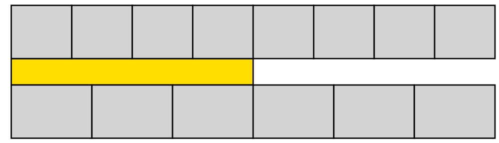
(d) How long is the red bar?

8. (a) How many fifths of a greystick is the same length as 12 twentieths of a greystick?



(b) How many fourths (or quarters) of a greystick is the same length as 15 twentieths of a greystick?

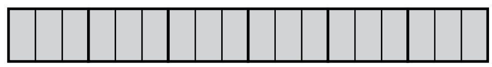
DESCRIBE THE SAME LENGTH IN DIFFERENT WAYS



Two fractions may describe the same length. You can see here that three sixths of a greystick is the same as four eighths of a greystick.

When two fractions describe the same portion we say they are **equivalent**.

1. (a) What can each small part on this greystick be called? ....



   (b) How many eighteenths is one sixth of the greystick? ....

   (c) How many eighteenths is one third of the greystick? ....

   (d) How many eighteenths is five sixths of the greystick? ....

2. (a) Write (in words) the names of four different fractions that are all equivalent to three quarters. You may look at the yellow greysticks on page 154 to help you.

   ....

   ....

   (b) Which equivalents for two thirds can you find on the yellow greysticks?

   ....

3. The information that 2 thirds is equivalent to 4 sixths, to 6 ninths and to 8 twelfths is written in the second row of the table below. Complete the other rows of the table in the same way. The diagrams on page 154 may help you.

thirds	fourths	fifths	sixths	eighths	ninths	tenths	twelfths	twentieths
1								
2	–	–	4	–	6	–	8	–
–	3							
–	–	1						
–	–	2						
–	–	3						
–	–	4						

4. Complete this table in the same way as the table in question 3.

fifths	tenths	fifteenths	twentieths	twenty-fifths	fiftieths	hundredths
1						
2						
3						
4						
5						
6						
7						

5. Draw on the greysticks below to show that 3 fifths and 9 fifteenths are equivalent. Draw freehand; you need not measure and draw accurately.

6. Complete these tables in the same way as the table in question 4.

eighths	sixteenths	24ths
1		
2		
3		
4		
5		
6		
7		
8		
9		

24ths	sixths	twelfths	18ths
	1		
	2		
	3		
	4		
	5		
	6		
	7		
	8		
	9		

7. (a) How much is five twelfths plus three twelfths? ....

   (b) How much is five twelfths plus one quarter? ....

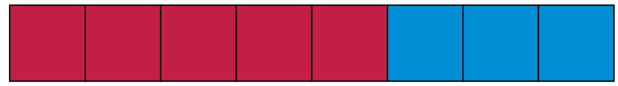
   (c) How much is five twelfths plus three quarters? ....

   (d) How much is one third plus one quarter? It may help if you work with the equivalent fractions in twelfths.

....

6.2 Different parts in different colours

This strip is divided into eight equal parts. Five eighths of this strip is red.

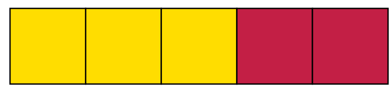


1. What part of the strip is blue?

   ....

2. What part of this strip is yellow?

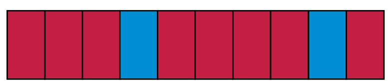
   ....



3. What part of the strip is red?

   ....

4. What part of this strip is coloured blue and what part is coloured red?



   ....

5. (a) What part of this strip is blue, what part is red and what part is white?

   ....

   (b) Express your answer differently with equivalent fractions.

   ....

6. A certain strip is not shown here. Two ninths of the strip is blue, and three ninths of the strip is green. The rest of the strip is red. What part of the strip is red?

   ....

7. What part of the strip below is yellow, what part is blue, and what part is red?

   ....

The number of parts in a fraction is called the **numerator** of the fraction. For example, the numerator in 5 sixths is 5.

The type of part in a fraction is called the **denominator**. It is the name of the parts that are being referred to and it is determined by the size of the part. For example, sixths is the denominator in 5 sixths.

$^5/_6$ is a short way to write 5 sixths.

We may also write $\frac{5}{6}$.

Even when we write $^5/_6$ or $\frac{5}{6}$, we still say "5 sixths".

$/_6$ and $\frac{\ }{6}$ are short ways to write *sixths*.

The numerator (number of parts) is written above the line of the fraction: $\frac{\textit{numerator}}{\dots}$

The denominator is indicated by a number written below the line: $\frac{\dots}{\textit{denominator}}$

To **enumerate** means "to find the number of".

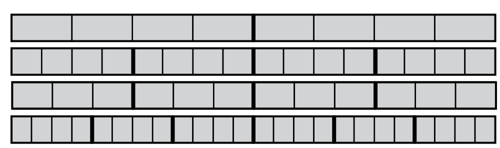
To **denominate** means "to give a name to".

8. Consider the fraction 3 quarters. It can be written as $\frac{3}{4}$.

   (a) Multiply both the numerator and the denominator by 2 to form a new fraction. Is the new fraction equivalent to $\frac{3}{4}$? You may check on the diagram below.

   ....

   ....



   (b) Multiply both the numerator and the denominator by 3 to form a new fraction. Is the new fraction equivalent to $\frac{3}{4}$?

   ....

   (c) Multiply both the numerator and the denominator by 4 to form a new fraction. Is the new fraction equivalent to $\frac{3}{4}$?

   ....

   (d) Multiply both the numerator and the denominator by 6 to form a new fraction. Is the new fraction equivalent to $\frac{3}{4}$?

   ....

6.3 Combining fractions

BIGGER AND SMALLER PARTS

Gertie was asked to solve this problem:

A team of road-builders built $\frac{8}{12}$ km of road in one week, and $\frac{10}{12}$ km in the next week. What is the total length of road that they built in the two weeks?

She thought like this to solve the problem:

*$\frac{8}{12}$ is **eight twelfths** and $\frac{10}{12}$ is **ten twelfths**, so altogether it is **eighteen twelfths**.*
I can write $\frac{18}{12}$ or "18 twelfths".
*I can also say 12 twelfths of a km is 1 km, so **18 twelfths** is **1 km and 6 twelfths of a km**.*
This I can write as $1\frac{6}{12}$. It is the same as $1\frac{1}{2}$ km.

Gertie was also asked the question: How much is $4\frac{5}{9} + 2\frac{7}{9}$?
She thought like this to answer it:

$4\frac{5}{9}$ is 4 wholes and 5 ninths, and $2\frac{7}{9}$ is 2 wholes and 7 ninths.
So altogether it is 6 wholes and 12 ninths. But 12 ninths is 9 ninths (1 whole) and 3 ninths, so I can say it is 7 wholes and 3 ninths.
I can write $7\frac{3}{9}$.

1. Would Gertie be wrong if she said her answer was $7\frac{1}{3}$?

   ....

2. Senthereng has $4\frac{7}{12}$ bottles of cooking oil. He gives $1\frac{5}{12}$ bottles to his friend Willem. How much oil does Senthereng have left?

   ....

3. Margaret has $5\frac{5}{8}$ bottles of cooking oil. She gives $3\frac{7}{8}$ bottles to her friend Naledi. How much oil does Margaret have left?

   ....

4. Calculate each of the following:

(a) $4\frac{2}{7} - 3\frac{6}{7}$

(b) $3\frac{6}{7} + \frac{3}{7}$

....

....

....

....

(c) $3\frac{6}{7} + 1\frac{4}{5}$

(d) $4\frac{3}{8} - 2\frac{4}{5}$

....

....

....

....

(e) $1\frac{3}{10} - \frac{2}{3}$

(f) $3\frac{5}{10} - 1\frac{1}{2}$

....

....

....

....

(g) $\frac{5}{8} + \frac{5}{8} + \frac{5}{8} + \frac{5}{8} + \frac{5}{8}$

(h) $6\frac{2}{5} + 2\frac{1}{4} - \frac{1}{2}$

....

....

....

....

(i) $\frac{5}{8} + \frac{5}{8} + \frac{5}{8} + \frac{5}{8} + \frac{5}{8} + \frac{5}{8} + \frac{5}{8} + \frac{5}{8} + \frac{5}{8} + \frac{5}{8} + \frac{5}{8} + \frac{5}{8} + \frac{5}{8}$

....

(j) $2\frac{4}{7} + 2\frac{4}{7} + 2\frac{4}{7} + 2\frac{4}{7} + 2\frac{4}{7} + 2\frac{4}{7} + 2\frac{4}{7} + 2\frac{4}{7}$

....

(k) $(4\frac{2}{7} + 1\frac{4}{7}) - 2\frac{1}{3}$

....

(l) $(2\frac{7}{10} + 3\frac{2}{5}) - (1\frac{2}{5} + 3\frac{7}{10})$

....

5. Neo's report had five chapters. The first chapter was $\frac{3}{4}$ of a page, the second chapter was $2\frac{1}{2}$ pages, the third chapter was $3\frac{3}{4}$ pages, the fourth chapter was 3 pages and the fifth chapter was $1\frac{1}{2}$ pages. How many pages was Neo's report in total?

....

6.4 Tenths and hundredths (percentages)

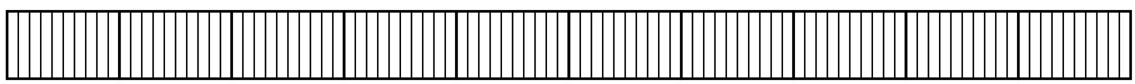
1. (a) 100 children each get 3 biscuits. How many biscuits is this in total?

   ....

   (b) 500 sweets are shared equally between 100 children. How many sweets does each child get?

   ....

2. The picture below shows a strip of licorice. The very small pieces can easily be broken off on the thin lines. How many very small pieces are shown on the picture?



   ....

3. Gatsha runs a spaza shop. He sells strips of licorice like the above for R2 each.

   (a) What is the cost of one very small piece of licorice, when you buy from Gatsha?

   ....

   (b) Jonathan wants to buy one fifth of a strip of licorice. How much should he pay?

   ....

   (c) Batseba eats 25 very small pieces. What part of a whole strip of licorice is this?

   ....

Each small piece of the above strip is **one hundredth** of the whole strip.

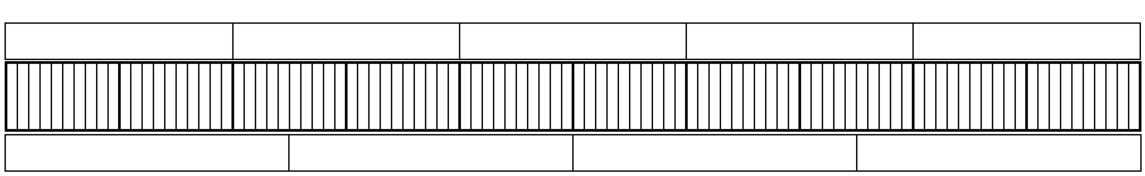
4. (a) Why can each small piece be called *one hundredth* of the whole strip?

   ....

   (b) How many hundredths is the same as one tenth of the strip?

   ....

Gatsha often sells parts of licorice strips to customers. He uses a "quarters marker" and a "fifths marker" to cut off the pieces correctly from full strips. His two markers are shown below, next to a full strip of licorice.



5. (a) How many hundredths is the same as two fifths of the whole strip?

....

(b) How many tenths is the same as $\frac{2}{5}$ of the whole strip?

....

(c) How many hundredths is the same as $\frac{3}{4}$ of the whole strip?

....

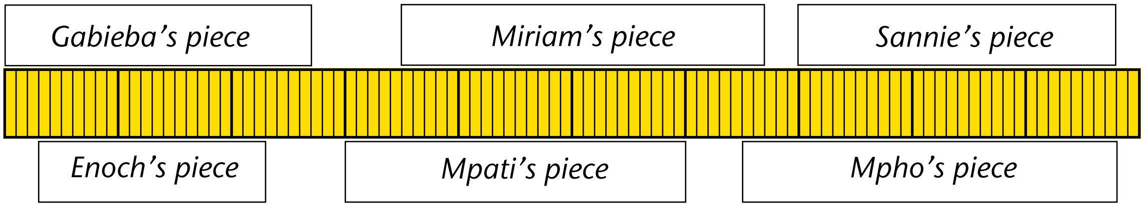
(d) Freddie bought $\frac{60}{100}$ of a strip. How many fifths of a strip is this?

....

(e) Jamey bought part of a strip for R1,60. What part of a strip did she buy?

....

6. Gatsha, the owner of the spaza shop, sold pieces of yellow licorice to different children. Their pieces are shown below.
How much (what part of a whole strip) did each of them get?



....

....

7. The yellow licorice shown above costs R2,40 (240 cents) for a strip. How much does each of the children have to pay? Round off the amounts to the nearest cent.

....

....

8. (a) How much is $\frac{1}{100}$ of 300 cents? (b) How much is $\frac{7}{100}$ of 300 cents?

....

(c) How much is $\frac{25}{100}$ of 300 cents? (d) How much is $\frac{1}{4}$ of 300 cents?

....

(e) How much is $\frac{40}{100}$ of 300 cents? (f) How much is $\frac{2}{5}$ of 300 cents?

....

9. Explain why your answers for questions 8(e) and 8(f) are the same.

....

Another word for **hundredth** is **per cent**.
Instead of saying
Miriam received **32 hundredths** of a licorice strip,
we can say
Miriam received **32 per cent** of a licorice strip.
The symbol for per cent is %.

10. How much is 80% of each of the following?

(a) R500 (b) R480 (c) R850 (d) R2 400

....

11. How much is 8% of each of the amounts in 10(a), (b), (c) and (d)?

....

12. How much is 15% of each of the amounts in 10(a), (b), (c) and (d)?

....

13. Building costs of houses increased by 20%. What is the new building cost for a house that previously cost R110 000 to build?

....

14. The value of a car decreases by 30% after one year. If the price of a new car is R125 000, what is the value of the car after one year?

....

15. Investigate which denominators of fractions can easily be converted to powers of 10.

....

6.5 Thousandths, hundredths and tenths

MANY EQUAL PARTS

1. In a camp for refugees, 50 kg of sugar must be shared equally between 1 000 refugees. How much sugar will each refugee get? Keep in mind that 1 kg is 1 000 g. You can give your answer in grams.

   ....

2. How much is each of the following?

   (a) one tenth of R6 000

   ....

   (b) one hundredth of R6 000

   ....

   (c) one thousandth of R6 000

   ....

   (d) ten hundredths of R6 000

   ....

   (e) 100 thousandths of R6 000

   ....

   (f) seven hundredths of R6 000

   ....

   (g) 70 thousandths of R6 000

   ....

   (h) seven thousandths of R6 000

   ....

3. Calculate.

   (a) $\frac{3}{10} + \frac{5}{8}$

   ....

   ....

   (b) $3\frac{3}{10} + 2\frac{4}{5}$

   ....

   ....

   (c) $\frac{3}{10} + \frac{7}{100}$

   ....

   ....

   (d) $\frac{3}{10} + \frac{70}{100}$

   ....

   ....

   (e) $\frac{3}{10} + \frac{7}{1\,000}$

   ....

   ....

   (f) $\frac{3}{10} + \frac{70}{1\,000}$

   ....

   ....

4. Calculate.

(a) $\frac{3}{10} + \frac{7}{100} + \frac{4}{1\,000}$

(b) $\frac{3}{10} + \frac{70}{100} + \frac{400}{1\,000}$

....

....

(c) $\frac{6}{10} + \frac{20}{100} + \frac{700}{1\,000}$

(d) $\frac{2}{10} + \frac{5}{100} + \frac{4}{1\,000}$

....

....

5. In each case investigate whether the statement is true or not, and give reasons for your final decision.

(a) $\frac{1}{10} + \frac{23}{100} + \frac{346}{1\,000} = \frac{6}{10} + \frac{3}{100} + \frac{46}{1\,000}$

....

....

(b) $\frac{1}{10} + \frac{23}{100} + \frac{346}{1\,000} = \frac{7}{10} + \frac{2}{100} + \frac{6}{1\,000}$

....

....

(c) $\frac{1}{10} + \frac{23}{100} + \frac{346}{1\,000} = \frac{6}{10} + \frac{7}{100} + \frac{6}{1\,000}$

....

....

(d) $\frac{676}{1\,000} = \frac{6}{10} + \frac{7}{100} + \frac{6}{1\,000}$

....

....

6.6 Fraction of a fraction

FORM PARTS OF PARTS

1. (a) How much is 1 fifth of R60? ....

   (b) How much is 3 fifths of R60? ....

2. How much is 7 tenths of R80?
   (You may first work out how much 1 tenth of R80 is.)

   ....

3. In the USA the unit of currency is the US dollar, in Britain it is the pound, in Western Europe the euro, and in Botswana the pula.

   (a) How much is 2 fifths of 20 pula? ....

   (b) How much is 2 fifths of 20 euro? ....

   (c) How much is 2 fifths of 12 pula? ....

4. Why was it so easy to calculate 2 fifths of 20, but difficult to calculate 2 fifths of 12?

   ....

There is a way to make it easy to calculate something like 3 fifths of R4. You just change the rands to cents!

5. Calculate each of the following. You may change the rands to cents to make it easier.

   (a) 3 eighths of R2,40 (b) 7 twelfths of R6

   ....

   (c) 2 fifths of R21 (d) 5 sixths of R3

   ....

6. You will now do some calculations about secret objects.

   (a) How much is 3 tenths of 40 secret objects?

   ....

   (b) How much is 3 eighths of 40 secret objects?

   ....

The secret objects in question 6 are fiftieths of a rand.

7. (a) How many fiftieths is 3 tenths of 40 fiftieths?

....

(b) How many fiftieths is 5 eighths of 40 fiftieths?

....

8. (a) How many twentieths of a kilogram is the same as $\frac{3}{4}$ of a kilogram? ....

(b) How much is one fifth of 15 rands? ....

(c) How much is one fifth of 15 twentieths of a kilogram? ....

(d) So, how much is one fifth of $\frac{3}{4}$ of a kilogram? ....

9. (a) How much is $\frac{1}{12}$ of 24 fortieths of some secret object? ....

(b) How much is $\frac{7}{12}$ of 24 fortieths of the secret object? ....

10. Do you agree that the answers for the previous question are 2 fortieths and 14 fortieths? If you disagree, explain why you disagree.

....

11. (a) How much is $\frac{1}{5}$ of 80? ....

(b) How much is $\frac{3}{5}$ of 80? ....

(c) How much is $\frac{1}{40}$ of 80? ....

(d) How much is $\frac{24}{40}$ of 80? ....

(e) Explain why $\frac{3}{5}$ of 80 is the same as $\frac{24}{40}$ of 80.

....

12. Look again at your answers for questions 9(b) and 11(e). How much is $\frac{7}{12}$ of $\frac{3}{5}$? Explain your answer.

....

....

The secret object in question 9 was an envelope with R160 in it.

After the work you did in questions 9, 10 and 11, you know that

- $\frac{24}{40}$ and $\frac{3}{5}$ are just two ways to describe the same thing, and
- $\frac{7}{12}$ of $\frac{3}{5}$ is therefore the same as $\frac{7}{12}$ of $\frac{24}{40}$.

It is easy to calculate $\frac{7}{12}$ of $\frac{24}{40}$: 1 twelfth of 24 is 2, so 7 twelfths of 24 is 14, so 7 twelfths of 24 fortieths is 14 fortieths.

$\frac{3}{8}$ of $\frac{2}{3}$ can be calculated in the same way. But 1 eighth of $\frac{2}{3}$ is a slight problem, so it would be better to use some equivalent of $\frac{2}{3}$. The equivalent should be chosen so that it is easy to calculate 1 eighth of it; so it would be nice if the numerator could be 8.

$\frac{8}{12}$ is equivalent to $\frac{2}{3}$, so instead of calculating $\frac{3}{8}$ of $\frac{2}{3}$ we may calculate $\frac{3}{8}$ of $\frac{8}{12}$.

13. (a) Calculate $\frac{3}{8}$ of $\frac{8}{12}$.

    ....

    (b) So, how much is $\frac{3}{8}$ of $\frac{2}{3}$?

    ....

14. In each case replace the second fraction by a suitable equivalent, and then calculate.

    (a) How much is $\frac{3}{4}$ of $\frac{5}{8}$?

    ....

    (b) How much is $\frac{7}{10}$ of $\frac{2}{3}$?

    ....

    (c) How much is $\frac{2}{3}$ of $\frac{1}{2}$?

    ....

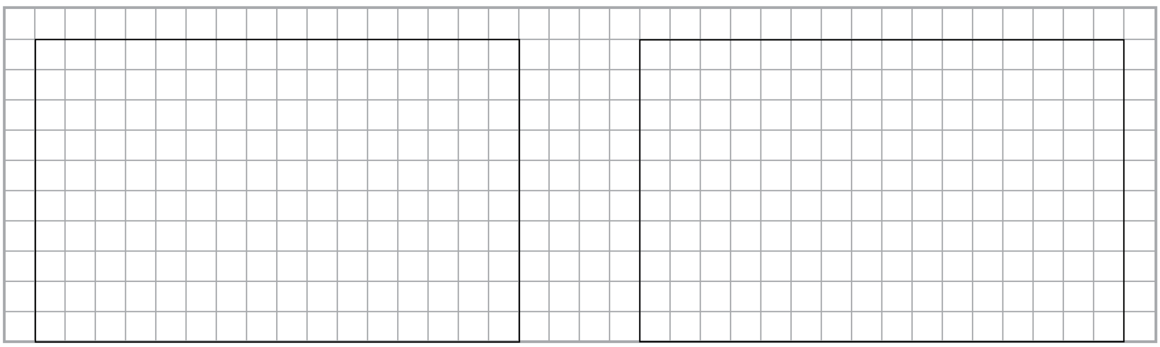
    (d) How much is $\frac{3}{5}$ of $\frac{3}{5}$?

    ....

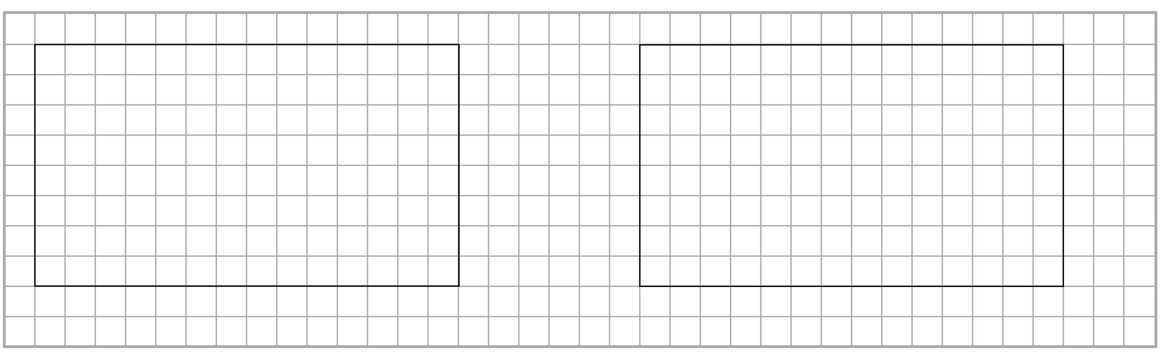
6.7 Multiplying with fractions

PARTS OF RECTANGLES, AND PARTS OF PARTS

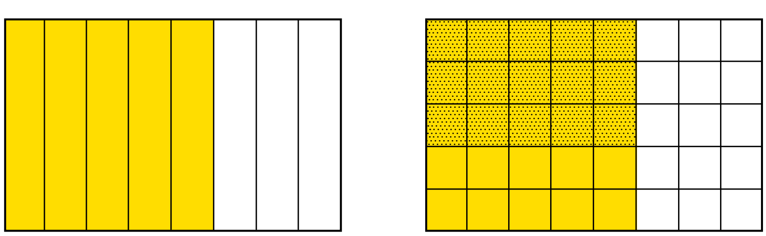
1. (a) Divide the rectangle on the left into eighths by drawing vertical lines. Lightly shade the left 3 eighths of the rectangle.
   (b) Divide the rectangle on the right into fifths drawing horizontal lines. Lightly shade the upper 2 fifths of the rectangle.



2. (a) Shade 4 sevenths of the rectangle on the left below.
   (b) Shade 16 twenty-eighths of the rectangle on the right below.



3. (a) What part of each big rectangle below is coloured yellow? ....
   (b) What part of the *yellow* part of the rectangle on the right is dotted? ....

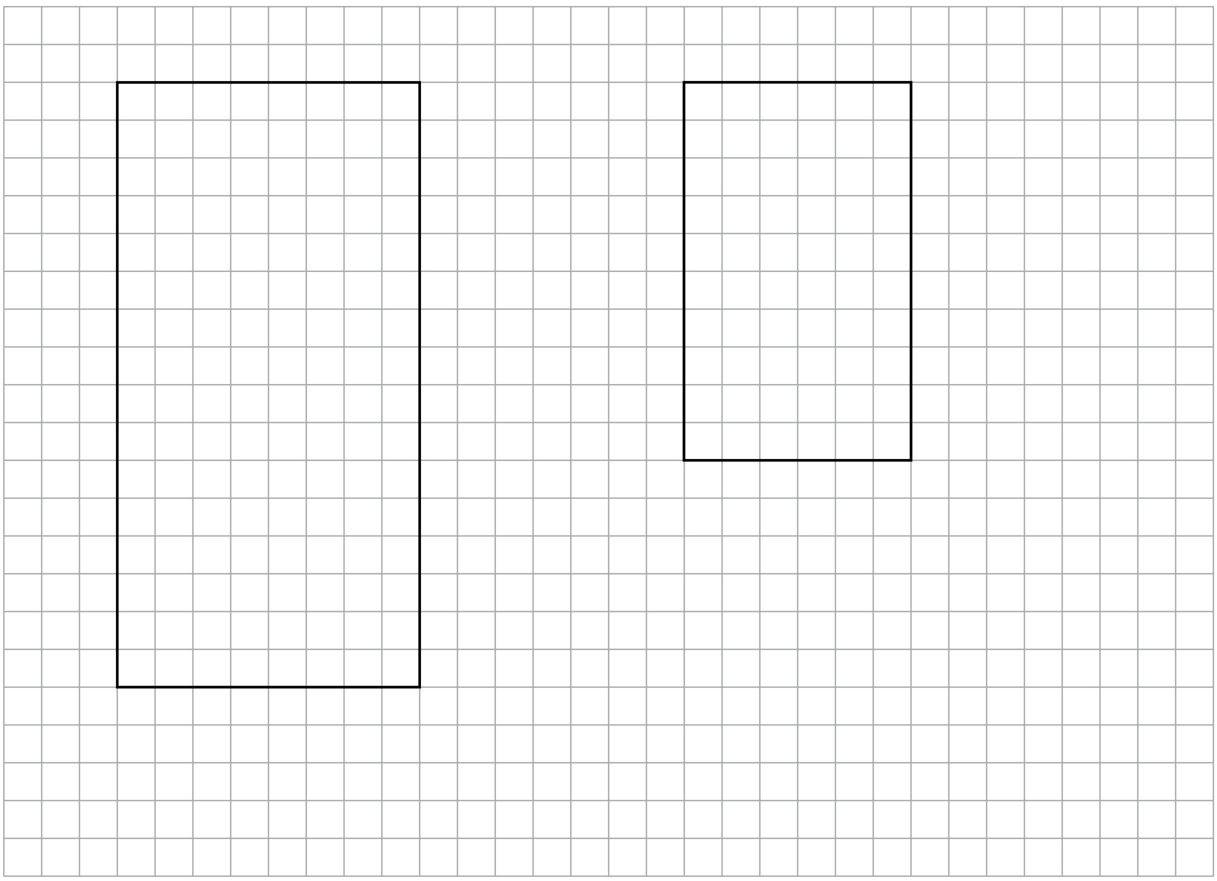


   (c) Into how many squares is the whole rectangle on the right divided? ....
   (d) What part of the whole rectangle on the right is yellow *and* dotted?

....

4. Make diagrams on the grid below to help you to figure out how much each of the following is:

   (a) $\frac{3}{4}$ of $\frac{5}{8}$ (b) $\frac{2}{3}$ of $\frac{4}{5}$



Here is something you can do with the fractions $\frac{3}{4}$ and $\frac{5}{8}$:
Multiply the two numerators and make this the numerator of a new fraction.
Also multiply the two denominators, and make this the denominator of a new fraction
$\frac{3 \times 5}{4 \times 8} = \frac{15}{32}$.

5. Compare the above with what you did in question 14(a) of section 6.6 and in question 4(a) at the top of this page. What do you notice about $\frac{3}{4}$ of $\frac{5}{8}$ and $\frac{3 \times 5}{4 \times 8}$?

6. (a) Alan has 5 heaps of 8 apples each. How many apples is that in total?

....

(b) Sean has 10 heaps of 6 quarter apples each. How many apples is that in total?

....

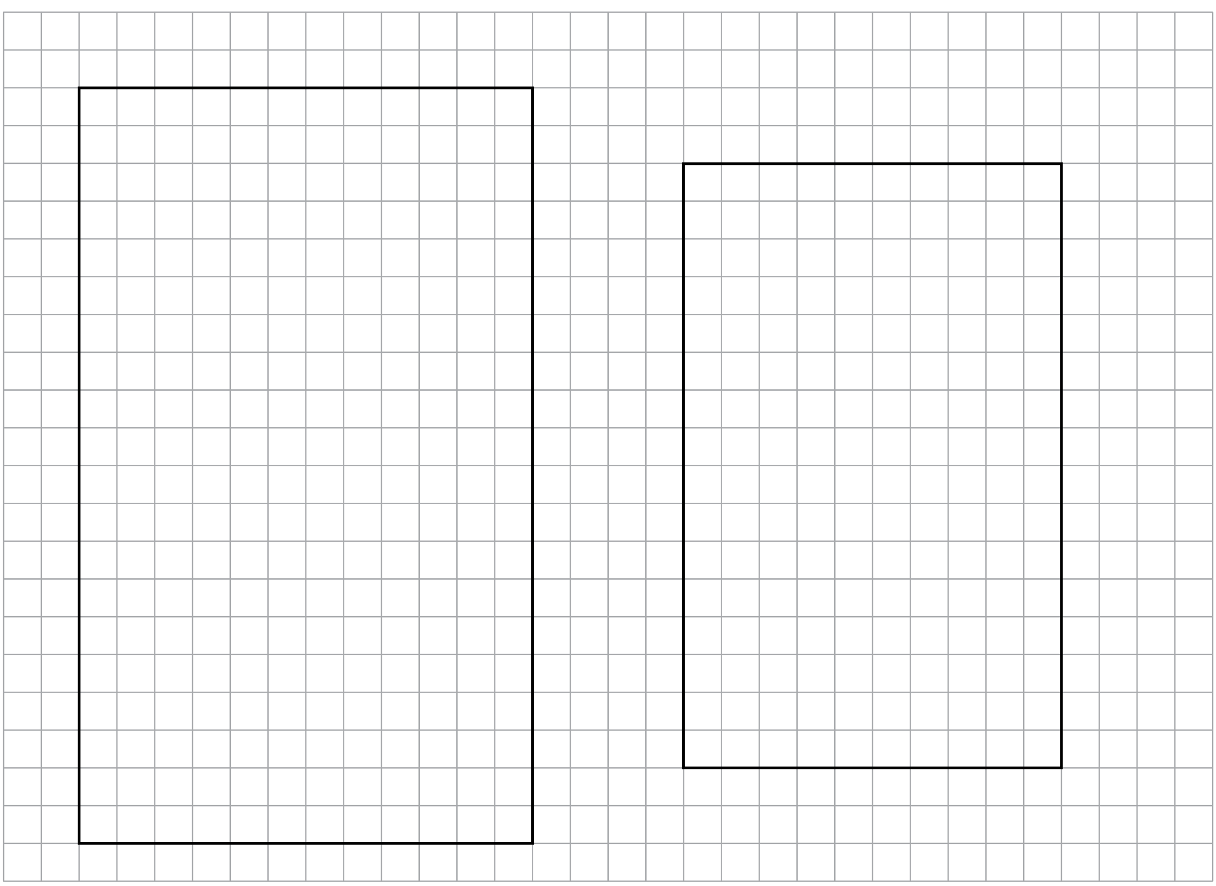
Instead of saying $\frac{5}{8}$ of R40 or $\frac{5}{8}$ of $\frac{2}{3}$ of a floor surface, we may say $\frac{5}{8} \times$ R40 or $\frac{5}{8} \times \frac{2}{3}$ of a floor surface.

7. Use the diagrams below to figure out how much each of the following is:

(a) $\frac{3}{10} \times \frac{5}{6}$

(b) $\frac{2}{5} \times \frac{7}{8}$

....



8. (a) Perform the calculations $\frac{\text{numerator} \times \text{numerator}}{\text{denominator} \times \text{denominator}}$ for $\frac{3}{10}$ and $\frac{5}{6}$ and compare the answer to your answer for question 7(a).

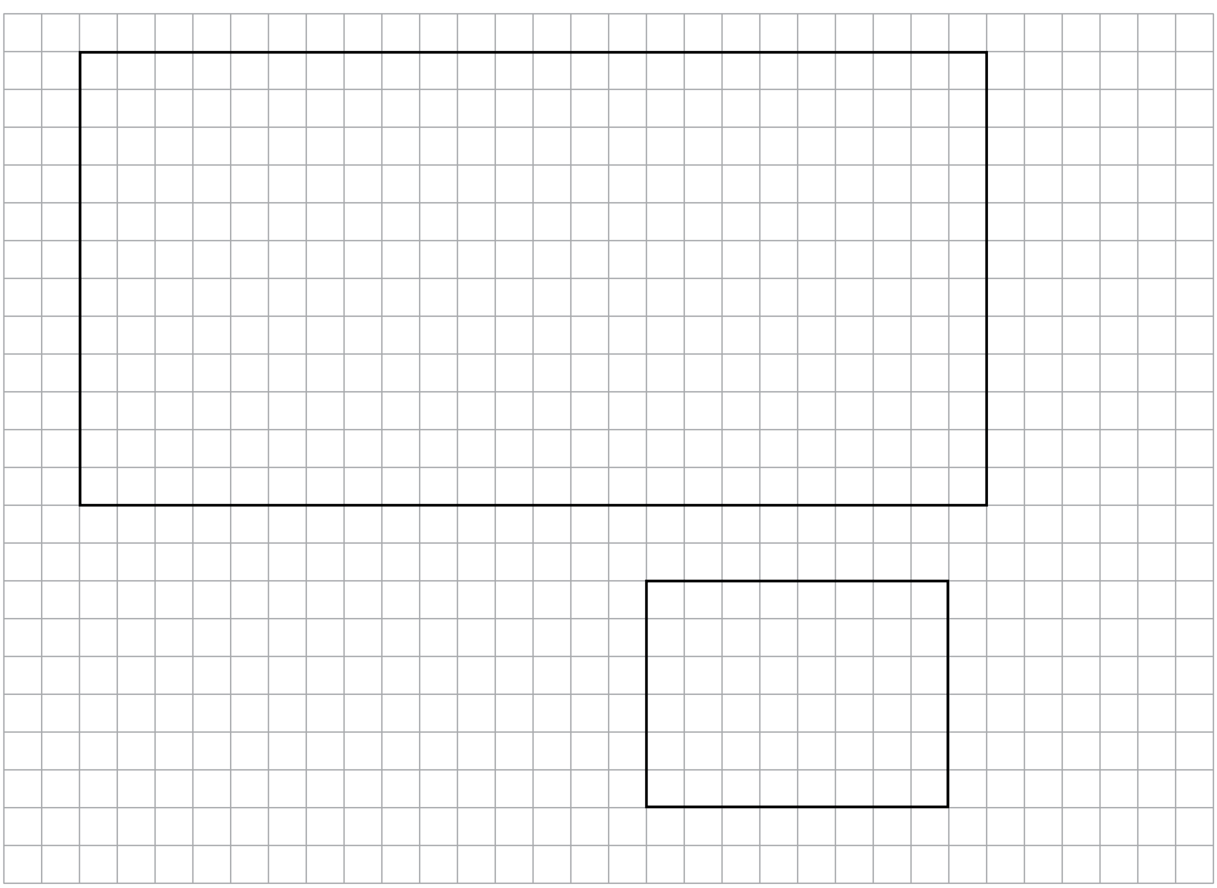
(b) Do the same for $\frac{2}{5}$ and $\frac{7}{8}$.

9. Perform the calculations $\frac{\text{numerator} \times \text{numerator}}{\text{denominator} \times \text{denominator}}$ for

(a) $\frac{5}{6}$ and $\frac{7}{12}$

(b) $\frac{3}{4}$ and $\frac{2}{3}$

10. Use the diagrams below to check whether the formula $\frac{\text{numerator} \times \text{numerator}}{\text{denominator} \times \text{denominator}}$ produces the correct answers for $\frac{5}{6} \times \frac{7}{12}$ and $\frac{3}{4} \times \frac{2}{3}$.



11. Calculate each of the following:

(a) $\frac{1}{2}$ of $\frac{1}{3}$ of R60 (b) $\frac{2}{7}$ of $\frac{2}{9}$ of R63 (c) $\frac{4}{3}$ of $\frac{2}{5}$ of R45

....

12. (a) John normally practises soccer for three quarters of an hour every day. Today he practised for only half his usual time. How long did he practise today?

....

....

(b) A bag of peanuts weighs $\frac{3}{8}$ of a kilogram. What does $\frac{3}{4}$ of a bag weigh?

....

(c) Calculate the mass of $7\frac{1}{2}$ packets of sugar if 1 packet has a mass of $\frac{3}{4}$ kg.

....

....

....

6.8 Ordering and comparing fractions

1. Order the following from the smallest to the biggest:

(a) $\frac{7}{16}$; $\frac{3}{8}$; $\frac{11}{24}$; $\frac{5}{12}$; $\frac{23}{48}$ (b) $\frac{703}{1\,000}$; $\frac{13}{20}$; $\frac{7}{10}$; 73%; $\frac{71}{100}$

....

....

2. Order the following from the biggest to the smallest:

(a) $\frac{41}{60}$; $\frac{19}{30}$; $\frac{7}{10}$; $\frac{11}{15}$; $\frac{17}{20}$ (b) $\frac{23}{24}$; $\frac{2}{3}$; $\frac{7}{8}$; $\frac{19}{20}$; $\frac{5}{6}$

....

....

3. Use the symbols = , > or < to make the following true:

(a) $\frac{7}{17}$ ☐ $\frac{21}{51}$ (b) $\frac{1}{17}$ ☐ $\frac{1}{19}$

WORKSHEET

1. Do the calculations given below. Rewrite each question in the common fraction notation. Then write the answer in words and in the common fraction notation.

   (a) 3 twentieths + 5 twentieths

   ....

   ....

   (b) 5 twelfths + 11 twelfths

   ....

   ....

   (c) 3 halves + 5 quarters

   ....

   ....

   (d) 3 fifths + 3 tenths

   ....

   ....

2. Complete the equivalent fractions.

   (a) $\frac{5}{7} = \frac{\square}{49}$

   (b) $\frac{9}{11} = \frac{\square}{33}$

   (c) $\frac{15}{10} = \frac{3}{\square}$

   (d) $\frac{1}{9} = \frac{4}{\square}$

   (e) $\frac{45}{18} = \frac{\square}{2}$

   (f) $\frac{4}{5} = \frac{\square}{35}$

3. Do the calculations given below. Rewrite each question in words. Then write the answer in words and in the common fraction notation.

   (a) $\frac{3}{10} + \frac{7}{30}$

   ....

   ....

   ....

   (b) $\frac{2}{5} + \frac{7}{12}$

   ....

   ....

   ....

   (c) $\frac{1}{100} + \frac{7}{10}$

   ....

   ....

   ....

   (d) $\frac{3}{5} - \frac{3}{8}$

   ....

   ....

   ....

   (e) $2\frac{3}{10} + 5\frac{9}{10}$

   ....

   ....

   ....

WORKSHEET

4. Joe earns R5 000 per month. His salary increases by 12%. What is his new salary?

 ....

5. Ahmed earned R7 500 per month. At the end of a certain month, his employer raised his salary by 10%. However, one month later his employer had to decrease his salary again by 10%. What was Ahmed's salary then?

 ....

 ....

6. Calculate each of the following and simplify the answer to its lowest form:

(a) $\frac{13}{20} - \frac{2}{5}$

 ....

(b) $3\frac{24}{100} - 1\frac{2}{10}$

 ....

(c) $5\frac{9}{11} - 2\frac{1}{4}$

 ....

(d) $\frac{2}{3} + \frac{4}{7}$

 ....

7. Evaluate.

(a) $\frac{1}{2} \times 9$ ....

(b) $\frac{3}{5} \times \frac{10}{27}$ ....

(c) $\frac{2}{3} \times 15$ ....

(d) $\frac{2}{3} \times \frac{3}{4}$ ....

8. Calculate.

(a) $2\frac{2}{3} \times 2\frac{2}{3}$

 ....

(b) $8\frac{2}{5} \times 3\frac{1}{3}$

 ....

(c) $(\frac{1}{3} + \frac{1}{2}) \times \frac{6}{7}$

 ....

(d) $\frac{2}{3} \times \frac{1}{2} \times \frac{3}{4}$

 ....

(e) $\frac{5}{6} + \frac{2}{3} \times \frac{1}{5}$

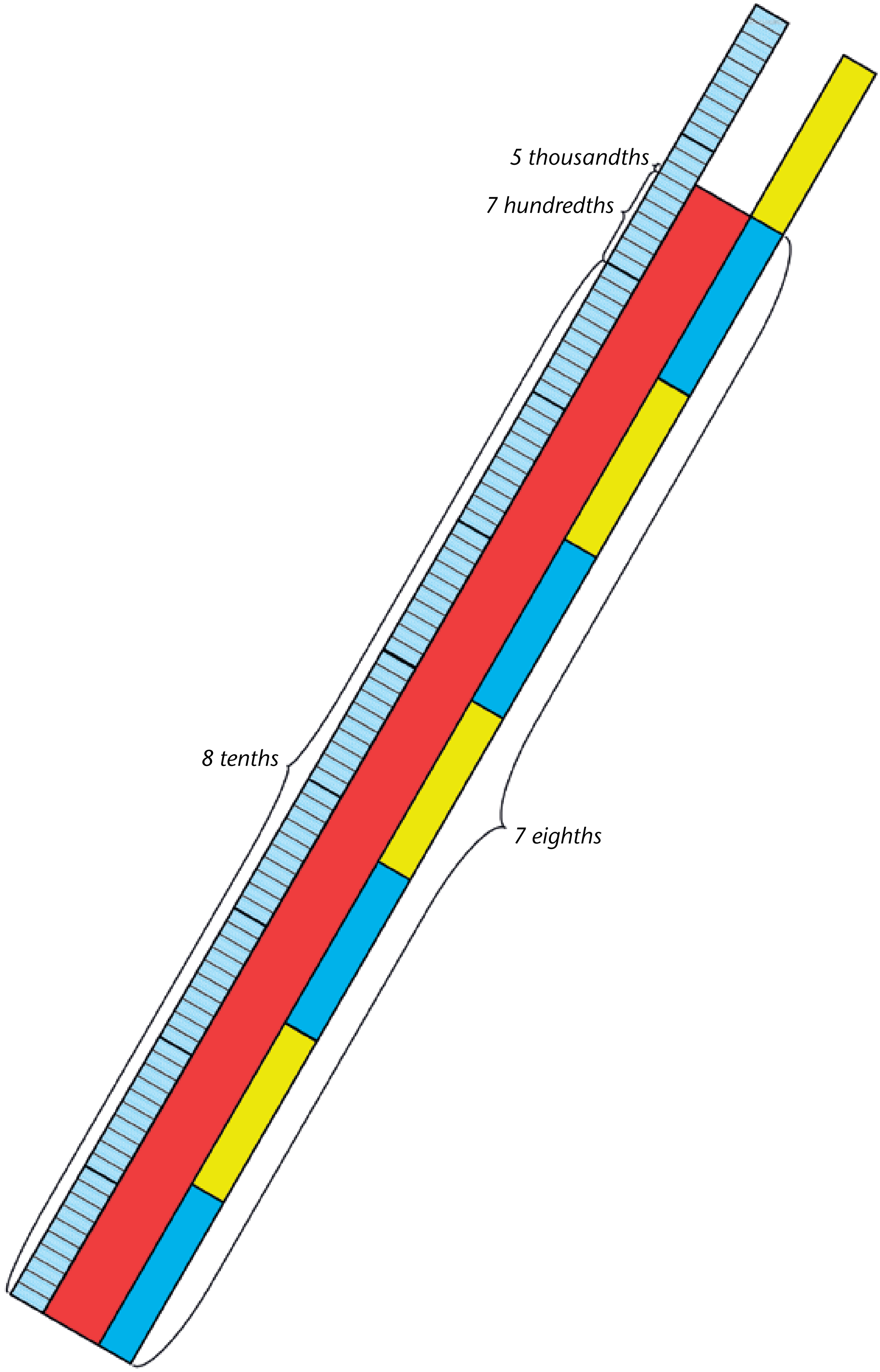
 ....

(f) $\frac{3}{4} - \frac{2}{5} \times \frac{5}{6}$

 ....

CHAPTER 7
The decimal notation for fractions

In this chapter you will learn more about decimal fractions and how they relate to common fractions and percentages. You will also learn to order and compare decimal fractions, and how to calculate with decimal fractions.


5 thousandths
7 hundredths
8 tenths
7 eighths

7 The decimal notation for fractions

7.1 Other symbols for tenths and hundredths

TENTHS AND HUNDREDTHS AGAIN ...

1. (a) What part of the rectangle below is coloured yellow?

   (b) What part of the rectangle is red? What part is blue? What part is green, and what part is not coloured?

0,1 is another way to write $\frac{1}{10}$ and
0,01 is another way to write $\frac{1}{100}$.
0,1 and $\frac{1}{10}$ are different notations for the same number.
$\frac{1}{10}$ is called the **(common) fraction notation** and 0,1 is called the **decimal notation**.

2. Write the answers for 1(a) and (b) in decimal notation.

3. 3 tenths and 7 hundredths of a rectangle is coloured red, and 2 tenths and 6 hundredths of the rectangle is coloured brown. What part of the rectangle (how many tenths and how many hundredths) is not coloured? Write your answer in fraction notation and in decimal notation.

4. On Monday, Steve ate 3 tenths and 7 hundredths of a strip of licorice. On Tuesday, Steve ate 2 tenths and 5 hundredths of a strip of licorice. How much licorice did he eat on Monday and Tuesday together? Write your answer in fraction notation and in decimal notation.

5. Lebogang's answer for question 4 is *5 tenths and 12 hundredths*. Susan's answer is *6 tenths and 2 hundredths*. Who is right, or are they both wrong?

....

The same quantity can be expressed in different ways in tenths and hundredths.

For example, *3 tenths and 17 hundredths* can be expressed as *2 tenths and 27 hundredths* or *4 tenths and 7 hundredths*.

All over the world, people have agreed to keep the number of hundredths in such statements below 10. This means that the normal way of expressing the above quantity is *4 tenths and 7 hundredths*.

Written in decimal notation, 4 tenths and 7 hundredths is 0,47. This is read as *nought comma four seven* and NOT *nought comma forty-seven*.

6. What is the decimal notation for each of the following numbers?

(a) $3\frac{7}{10}$ (b) $4\frac{19}{100}$ (c) $\frac{47}{10}$ (d) $\frac{4}{100}$

....

... AND THOUSANDTHS

0,001 is another way of writing $\frac{1}{1\,000}$.

1. What is the decimal notation for each of the following?

(a) $\frac{7}{1\,000}$ (b) $\frac{9}{1\,000}$ (c) $\frac{147}{1\,000}$ (d) $\frac{999}{1\,000}$

....

2. Write the following numbers in the decimal notation:

(a) $2 + \frac{3}{10} + \frac{7}{100} + \frac{4}{1\,000}$ (b) $12 + \frac{1}{10} + \frac{4}{1\,000}$

....

(c) $2 + \frac{4}{1\,000}$ (d) $67\frac{123}{1\,000}$

....

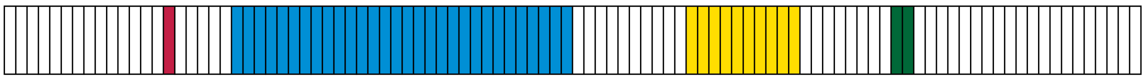
(e) $34\frac{61}{1\,000}$ (f) $654\frac{3}{1\,000}$

....

7.2 Percentages and decimal fractions

HUNDREDTHS, PERCENTAGES AND DECIMALS

1. The rectangle below is divided into small parts.



   (a) How many of these small parts are there in the rectangle? And in one tenth of the rectangle?

   ....

   (b) What part of the rectangle is blue? What part is green? What part is red?

   ....

Instead of *6 hundredths*, you may say 6 *per cent*. It means the same.
10 per cent of the rectangle above is yellow.

2. Use the word "per cent" to say what part of the rectangle is green. What part is red?

   ....

3. What percentage of the rectangle is blue? What percentage is white?

   ....

We do not say: "How many per cent of the rectangle is green?" We say: "What percentage of the rectangle is green?"

The symbol % is used for "per cent".
Instead of writing "17 per cent", you may write 17%.
Per cent means *hundredths*. The symbol % is a bit like the symbol $\frac{\ }{100}$.

4. (a) How much is 1% of R400? (In other words: How much is $\frac{1}{100}$ or 0,01 of R400?)

   ....

   (b) How much is 37% of R400?

   ....

   (c) How much is 37% of R700?

   ....

5. (a) 25 apples are shared equally between 100 people. How much apple does each person get? Write your answer as a common fraction and as a decimal fraction.

....

(b) How much is 1% (one hundredth) of 25?

....

(c) How much is 8% of 25?

....

(d) How much is 8% of 50? And how much is 0,08 of 50?

....

0,37 and 37% and $\frac{37}{100}$ are different symbols for the same thing: *37 hundredths.*

6. Express each of the following in three ways:
   - in the *decimal notation*,
   - in the *percentage notation* and
   - if possible, in the *common fraction notation, using hundredths*.

(a) 3 tenths

....

(b) 7 hundredths

....

(c) 37 hundredths

....

(d) 7 tenths

....

(e) 3 quarters

....

(f) 7 eighths

....

7. (a) How much is 3 tenths of R200 and 7 hundredths of R200 altogether?

....

(b) How much is $\frac{37}{100}$ of R200?

....

(c) How much is 0,37 of R200?

....

(d) And how much is 37% of R200?

....

8. Express each of the following in three ways:
   - in the *decimal notation*,
   - in the *percentage notation* and
   - in the *common fraction notation, using hundredths*.

   (a) 20 hundredths (b) 50 hundredths

   ....

   (c) 25 hundredths (d) 75 hundredths

   ....

9. (a) Jan eats a quarter of a watermelon. What percentage of the watermelon is this?

   ....

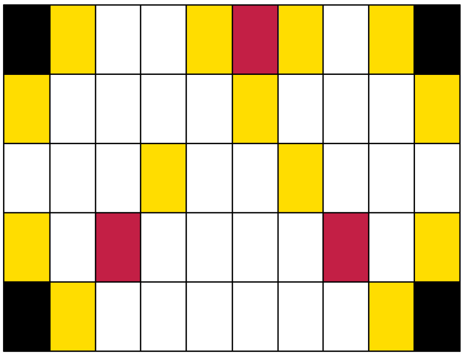
   (b) Sibu drinks 75% of the milk in a bottle. What fraction of the milk is this?

   ....

   (c) Jeminah uses 0,75 (7 tenths and 5 hundredths) of the paint in a tin. What percentage of the paint does she use?

   ....

10. The floor of a large room is shown alongside. What part of the floor is covered in each of the four colours? Express your answer in four ways:



   (a) in the *common fraction notation, using hundredths*,
   (b) in the *decimal notation*,
   (c) in the *percentage notation*, and
   (d) if possible, *in the common fraction notation, as tenths* ***and*** *hundredths* (for example $\frac{3}{10} + \frac{4}{100}$).

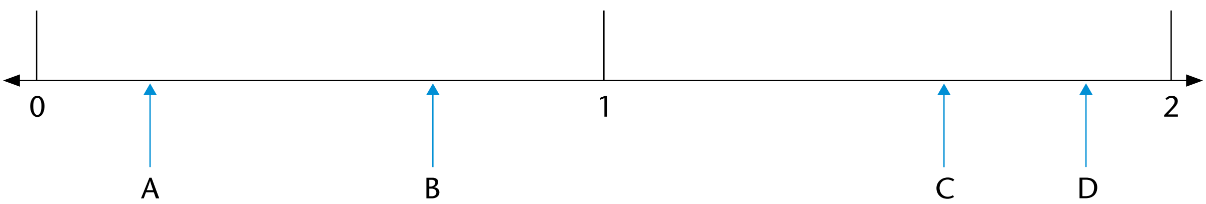
	(a)	(b)	(c)	(d)
white				
red				
yellow				
black				

7.3 Decimal measurements

MEASURING ON A NUMBER LINE

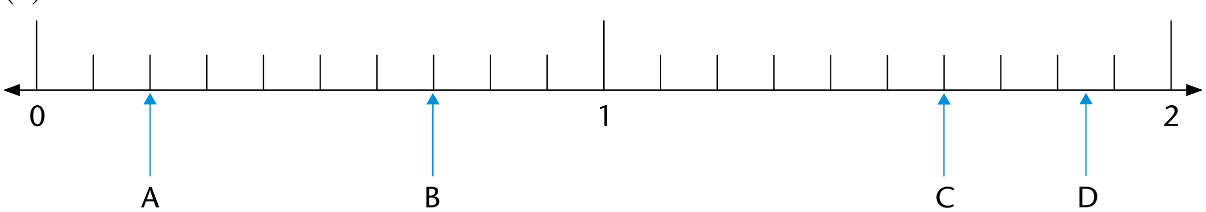
1. Read the lengths at the marked points (A to D) on the number lines. Give your answers as accurate as possible in decimal notation.

(a)



....

(b)

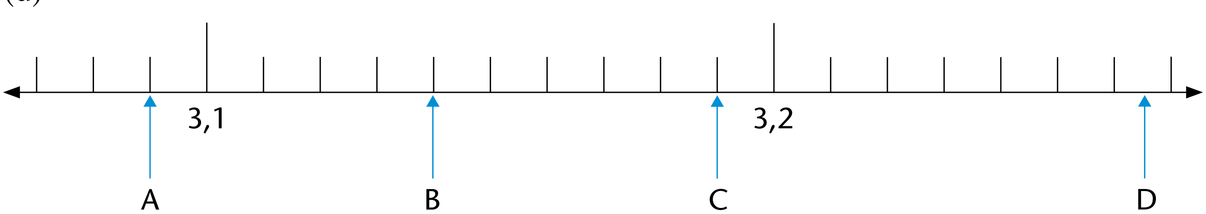


....

(c)

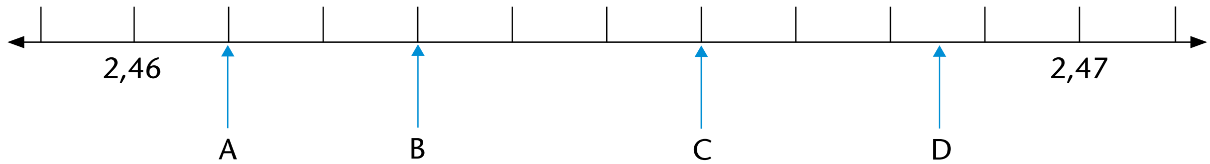
....

(d)

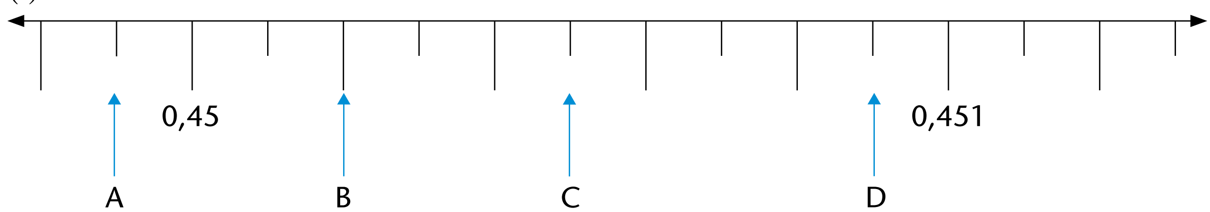


....

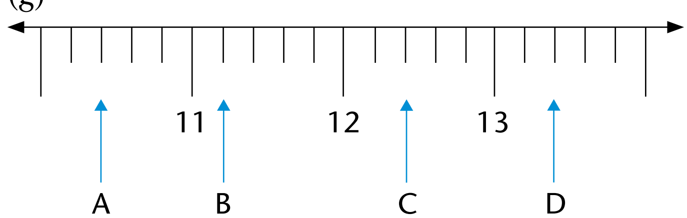
(e)



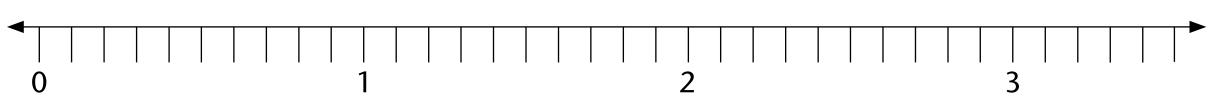
(f)



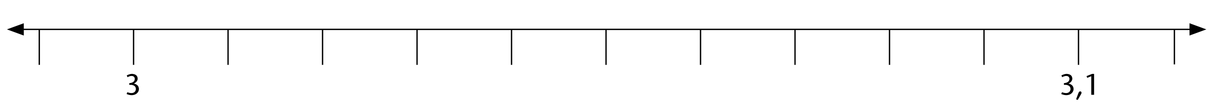
(g)



2. Show the following numbers on the number line below:
   (a) 0,6 (b) 1,2 (c) 1,85 (d) 2,3
   (e) 2,65 (f) 3,05 (g) 0,08



3. Show the following numbers on the number line below:
   (a) 3,06 (b) 3,08 (c) 3,015
   (d) 3,047 (e) 3,005



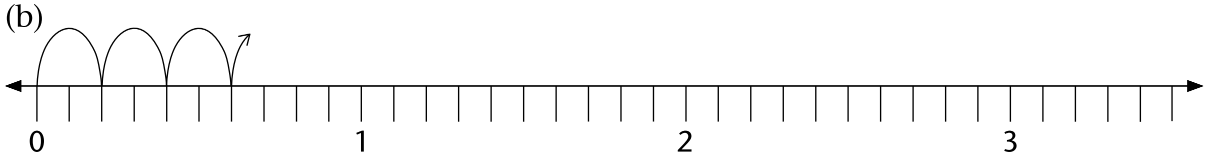
7.4 More decimal concepts

DECIMAL JUMPS

Write the next ten numbers in the number sequences and show your number sequences, as far as possible, on the number lines.

1. (a) 0,2; 0,4; 0,6; ....

   (b)

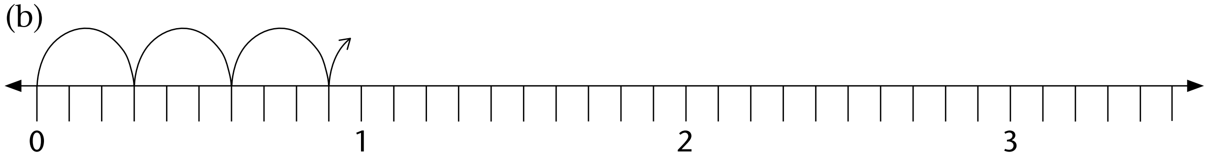


   (c) How many 0,2s are there in 1? ....

   (d) Write 0,2 as a common fraction. ....

2. (a) 0,3; 0,6; 0,9; ....

   (b)

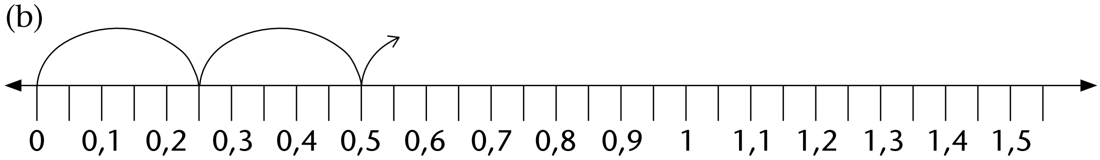


   (c) How many 0,3s are there in 3? ....

   (d) Write 0,3 as a common fraction. ....

3. (a) 0,25; 0,5; ....

   (b)



   (c) How many 0,25s are there in 1? ....

   (d) Write 0,25 as a common fraction. ....

A calculator can be programmed to do the same operation over and over again.

For example, press 0,1 $\boxed{+}$ $\boxed{=}$ (do not press CLEAR or any other operation). Press the $\boxed{=}$ key repeatedly and see what happens.

The calculator counts in 0,1s.

4. You can check your answers for questions 1 to 3 with a calculator. Program the calculator to help you.

5. Write the next five numbers in the number sequences:
   (a) 9,3; 9,2; 9,1; ....
   (b) 0,15; 0,14; 0,13; 0,12; ....

6. Check your answers with a calculator. Program the calculator to help you.

PLACE VALUE

1. Write each of the following as one number:

(a) 2 + 0,5 + 0,07	(b) 2 + 0,5 + 0,007
....	....
(c) 2 + 0,05 + 0,007	(d) 5 + 0,4 + 0,03 + 0,001
....	....
(e) 5 + 0,04 + 0,003 + 0,1	(f) 5 + 0,004 + 0,3 + 0,01
....	....

We can write 3,784 in expanded notation as 3,784 = 3 + 0,7 + 0,08 + 0,004.
We can also name these parts as follows:

- the 3 represents the **units**
- the 7 represents the **tenths**
- the 8 represents the **hundredths**
- the 4 represents the **thousandths**

> We say: the **value** of the 7 is 7 tenths but the **place value** of the 7 is tenths, because any digit **in that place** will represent the number of tenths.
>
> For example, in 2,536 the **value** of the 3 is 0,03, and its **place value** is hundredths, because the value of the **place where it stands** is hundredths.

2. Now write the value (in decimal fractions) and the place value of each of the underlined digits.

(a) 2,3$\underline{4}$5	(b) 4,67$\underline{8}$	(c) 1,$\underline{9}$53
....	....	....
(d) 34,$\underline{8}$56	(e) 564,3$\underline{4}$	(f) 0,98$\underline{7}$
....	....	....

7.5 Ordering and comparing decimal fractions

FROM BIGGEST TO SMALLEST AND SMALLEST TO BIGGEST

1. Order the following numbers from biggest to smallest. Explain your method.

   0,8   0,05   0,5   0,15   0,465   0,55   0,75   0,4   0,62

   ....

   ....

   ....

   ....

2. Below are the results of some of the 2012 London Olympic events. In each case, order them from first to last place. Use the column provided.

   (a) Women: Long jump – Final

Name	Country	Distance	Place
Anna Nazarova	RUS	6,77 m	
Brittney Reese	USA	7,12 m	
Elena Sokolova	RUS	7,07 m	
Ineta Radevica	LAT	6,88 m	
Janay DeLoach	USA	6,89 m	3rd
Lyudmila Kolchanova	RUS	6,76 m	

   (b) Women: 400 m hurdles – Final

Name	Country	Time	Place
Georganne Moline	USA	53,92 s	
Kaliese Spencer	JAM	53,66 s	4th
Lashinda Demus	USA	52,77 s	
Natalya Antyukh	RUS	52,70 s	
T'erea Brown	USA	55,07 s	
Zuzana Hejnová	CZE	53,38 s	

(c) Men: 110 m hurdles – Final

Name	**Country**	**Time**	**Place**
Aries Merritt	USA	12,92 s	
Hansle Parchment	JAM	13,12 s	
Jason Richardson	USA	13,04 s	
Lawrence Clarke	GBR	13,39 s	
Orlando Ortega	CUB	13,43 s	
Ryan Brathwaite	BAR	13,40 s	

(d) Men: Javelin – Final

Name	**Country**	**Distance**	**Place**
Andreas Thorkildsen	NOR	82,63 m	
Antti Ruuskanen	FIN	84,12 m	
Keshorn Walcott	TRI	84,58 m	
Oleksandr Pyatnytsya	UKR	84,51 m	
Tero Pitkämäki	FIN	82,80 m	
Vítezslav Veselý	CZE	83,34 m	

3. In each case, give a number that falls between the two numbers.
(This means you may give *any* number that falls anywhere between the two numbers.)

(a) 3,5 and 3,7 (b) 3,9 and 3,11 (c) 3,1 and 3,2

....

4. How many numbers are there between 3,1 and 3,2?

....

5. Fill in <, > or =.

(a) 0,4 ☐ 0,52 (b) 0,4 ☐ 0,32

(c) 2,61 ☐ 2,7 (d) 2,4 ☐ 2,40

(e) 2,34 ☐ 2,567 (f) 2,34 ☐ 2,251

7.6 Rounding off

Just as whole numbers can be rounded off to the nearest 10, 100 or 1 000, decimal fractions can be rounded off to the nearest whole number or to one, two, three etc. digits after the comma. A decimal fraction is rounded off to the number whose value is closest to it. Therefore 13,24 rounded off to one decimal place is 13,2 and 13,26 rounded off to one decimal place is 13,3. A decimal ending in a 5 is an equal distance from the two numbers to which it can be rounded off. Such decimals are rounded off to the biggest number, so 13,15 rounded off to one decimal place becomes 13,2.

SAYING IT NEARLY BUT NOT QUITE

1. Round each of the following numbers off to the nearest whole number:
   7,6 18,3 204,5 1,89 0,9 34,7 11,5 0,65

   ....

2. Round each of the following numbers off to one decimal place:
   7,68 18,93 21,47 0,643 0,938 1,44 3,81

   ....

3. Round each of the following numbers off to two decimal places:
   3,432 54,117 4,809 3,762 4,258 10,222 9,365 299,996

   ....

ROUND OFF TO HELP YOU CALCULATE

1. John and three of his brothers sell an old bicycle for R44,65. How can the brothers share the money fairly?

   ....

   ....

2. A man buys 3,75 m of wood at R11,99 per metre. What does the wood cost him?

   ....

3. Estimate the answers of each of the following by rounding off the numbers:
   (a) $89,3 \times 3,8$ (b) $227,3 + 71,8 - 28,6$

   ....

7.7 Addition and subtraction with decimal fractions

MENTAL CALCULATIONS

1. Complete the number chain.

34,123	→	+ 20	→	54,123	→		→	454,123	→		→	454,023
												↓
												↓
422,011	←		←	452,011	←		←	452,021	←		←	452,023
↓												
↓												
222,011	→		→	222,211	→		→	222,231	→		→	222,232
												↓
												↓
222,489	←		←	222,482	←		←	222,422	←		←	222,222

When you add or subtract decimal fractions, you can change them to common fractions to make the calculation easier.

For example:

$0,4 + 0,5$

$= \frac{4}{10} + \frac{5}{10}$

$= \frac{9}{10}$

$= 0,9$

2. Calculate each of the following:

(a) $0,7 + 0,2$ ....

(b) $0,7 + 0,4$ ....

(c) $1,3 + 0,8$ ....

(d) $1,35 + 0,8$ ....

(e) $0,25 + 0,7$ ....

(f) $0,25 + 0,07$ ....

(g) $3 - 0,1$ ....

(h) $3 - 0,01$ ....

(i) $2,4 - 0,5$ ....

SOME REAL-LIFE PROBLEMS

1. The owner of an internet café looks at her bank statement at the end of the day. She finds the following amounts paid into her account: R281,45; R39,81; R104,54 and R9,80. How much money was paid into her account on that day?

 ....

2. At the beginning of a journey the odometer in a car reads: 21589,4. At the end of the journey the odometer reads: 21763,7. What distance was covered?

 ....

3. At an athletics competition, an athlete runs the 100 m race in 12,8 seconds. The announcer says that the athlete has broken the previous record by 0,4 seconds. What was the previous record?

 ....

4. In a surfing competition five judges give each contestant a mark out of 10. The highest and the lowest marks are ignored and the other three marks are totalled. Work out each contestant's score and place the contestants in order from first to last.

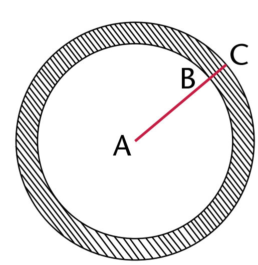
A:	7,5	8	7	8,5	7,7	B:	8,5	8,5	9,1	8,9	8,7
C:	7,9	8,1	8,1	7,8	7,8	D:	8,9	8,7	9	9,3	9,1

 ....

 ....

 ....

5. A pipe is measured accurately. AC = 14,80 mm and AB = 13,97 mm.
 How thick is the pipe (BC)?



 ....

 ....

6. Mrs Mdlankomo buys three packets of mincemeat.
 The packets weigh 0,356 kg, 1,201 kg and 0,978 kg respectively.
 What do they weigh together?

 ....

7.8 Multiplication and decimal fractions

THE POWER OF TEN

1. (a) Complete the multiplication table.

×	1 000	100	10	1	0,1	0,01	0,001
6	6 000		60			0,06	
6,4		640					
0,5					0,05		
4,78	4 780		47,8				
41,2	41 200						

(b) Is it correct to say that "multiplication makes bigger"? When does multiplication make bigger?

....

(c) Formulate rules for multiplying with 10; 100; 1 000; 0,1; 0,01 and 0,001. Can you explain the rules?

....

....

....

....

(d) Now use your rules to calculate each of the following:

0,5 × 10   0,3 × 100   0,42 × 10   0,675 × 100

....

2. (a) Complete the division table.

÷	0,001	0,01	0,1	1	10	100	1 000
6				6	0,6	0,06	
6,4			64	6,4			
0,5						0,005	
4,78			47,8				
41,2		4 120					

(b) Is it correct to say that "division makes smaller"? When does division make smaller?

....

(c) Formulate rules for dividing with 10; 100; 1 000; 0,1; 0,01 and 0,001. Can you explain the rules?

....

....

....

....

(d) Now use your rules to calculate each of the following:

0,5 ÷ 10   0,3 ÷ 100   0,42 ÷ 10

....

3. Complete the following:
   (a) Multiplying with 0,1 is the same as dividing by ....
   (b) Dividing by 0,1 is the same as multiplying by ....

   Now discuss it with a partner or explain to him or her why this is so.

4. Fill in the missing numbers:

1,23456	→	× 10	→	12,3456	→		→	123,456	→		→	12 345,6
												↓
												↓
1 234,56	←		←	12 345,6	←		←	123 456	←		←	1 234 560
↓												
↓												
123,456	→		→	1,23456	→		→	0,123456	→		→	123 456

What does multiplying a decimal number with a whole number mean?

What does something like 4 × 0,5 mean?

What does something like 0,5 × 4 mean?

4 × 0,5 means 4 groups of $\frac{1}{2}$, which is $\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2}$, which is 2.

0,5 × 4 means $\frac{1}{2}$ of 4, which is 2.

A real-life example where we would find this is:

$$\begin{aligned} 6 \times 0{,}42 \text{ kg} &= 6 \times \frac{42}{100} \\ &= (6 \times 42) \div 100 \\ &= 252 \div 100 \\ &= 2{,}52 \text{ kg} \end{aligned}$$

What really happens is that we convert 6 × 0,42 to the product of two whole numbers, do the calculation and then convert the answer to a decimal fraction again (÷ 100).

MULTIPLYING DECIMALS WITH WHOLE NUMBERS

1. Calculate each of the following. Use fraction notation to help you.

   (a) 0,3 × 7 (b) 0,21 × 91 (c) 8 × 0,4

   ....

   ....

   ....

2. Estimate the answers to each of the following and then calculate:

   (a) 0,4 × 7 (b) 0,55 × 7

   ....

   (c) 12 × 0,12 (d) 0,601 × 2

   ....

3. Make a rule for multiplying with decimals. Explain your rule to a partner.

   ....

   ....

What does multiplying a decimal with a decimal mean?

For example, what does 0,32 × 0,87 mean?

If you buy 0,32 m of ribbon and each metre costs R0,87, you can write it as 0,32 × 0,87.

$$0{,}32 \times 0{,}87 = \frac{32}{100} \times \frac{87}{100} \quad \text{[Write as common fractions]}$$

$$= \frac{32 \times 87}{10\,000} \quad \text{[Multiplication of two fractions]}$$

$$= \frac{2\,784}{10\,000} \quad \text{[The product of the whole numbers } 32 \times 87\text{]}$$

$$= 0{,}2784 \quad \text{[Convert to a decimal by dividing the product by } 10\,000\text{]}$$

The product of two decimals is thus converted to the product of whole numbers and then converted back to a decimal.

The product of two decimals and the product of two whole numbers with the same digits differ only in terms of the place value of the products, in other words the position of the decimal comma. It can also be determined by estimating and checking.

MULTIPLYING DECIMALS WITH DECIMALS

1. Calculate each of the following. Use fraction notation to help you.

   (a) 0,6 × 0,4 (b) 0,06 × 0,4 (c) 0,06 × 0,04

   ....

   ....

   ....

Mandla uses this method to multiply decimals with decimals:

$$0{,}84 \times 0{,}6 = (84 \div 100) \times (6 \div 10)$$
$$= (84 \times 6) \div (100 \times 10)$$
$$= 504 \div 1\,000$$
$$= 0{,}504$$

2. Calculate the following using Mandla's method:

   (a) 0,4 × 0,7 (b) 0,4 × 7 (c) 0,04 × 0,7

   ....

   ....

   ....

   ....

7.9 Division and decimal fractions

Look carefully at the following three methods of calculation:

1. 0,6 ÷ 2 = 0,3 [6 tenths ÷ 2 = 3 tenths]

2. 12,4 ÷ 4 = 3,1 [(12 units + 4 tenths) ÷ 4]
   = (12 units ÷ 4) + (4 tenths ÷ 4)
   = 3 units + 1 tenth
   = 3,1

3. 2,8 ÷ 5 = 28 tenths ÷ 5
   = 25 tenths ÷ 5 and 3 tenths ÷ 5
   = 5 tenths and (3 tenths ÷ 5) [3 tenths cannot be divided by 5]
   = 5 tenths and (30 hundredths ÷ 5) [3 tenths = 30 hundredths]
   = 5 tenths and 6 hundredths
   = 0,56

DIVIDING DECIMALS BY WHOLE NUMBERS

1. Complete.
   (a) 8,4 ÷ 2
   = (8 .... + 4 tenths) ÷ 2
   = (8 .... ÷ 2) + (..............)
   = 4 .... + ... tenths
   = ...
   (b) 3,4 ÷ 4
   = (3 units + 4 tenths) ÷ 4
   = (32 .... + 20 ....) ÷ 4
   = (........... ÷ 4) + (................ ÷ 4)
   = ... + ... hundredths
   = ...

2. Calculate each of the following in the shortest possible way:
   (a) 0,08 ÷ 4 (b) 14,4 ÷ 12

   ....

(c) 8,4 ÷ 7

(d) 4,5 ÷ 15

....

(e) 1,655 ÷ 5

(f) 0,225 ÷ 25

....

3. A grocer buys 15 kg of bananas for R99,90. What do the bananas cost per kilogram?

....

....

4. Given 26,8 ÷ 4 = 6,7. Write down the answers to the following without calculating:

(a) 268 ÷ 4

(b) 0,268 ÷ 4

(c) 26,8 ÷ 0,4

....

5. Given 128 ÷ 8 = 16. Write down the answers to the following without calculating:

(a) 12,8 ÷ 8

(b) 1,28 ÷ 8

(c) 1,28 ÷ 0,8

....

6. Sue pays R18,60 for 0,6 metres of material. What does one metre of material cost?

....

7. John buys 0,45 m of chain. The chain costs R20 per metre. What does John pay for the chain?

....

8. You may use a calculator for this question.

Anna buys a packet of mincemeat. It weighs 0,215 kg. The price for the mincemeat is R42,95 per kilogram. What does she pay for her packet of mincemeat? (Give a sensible answer.)

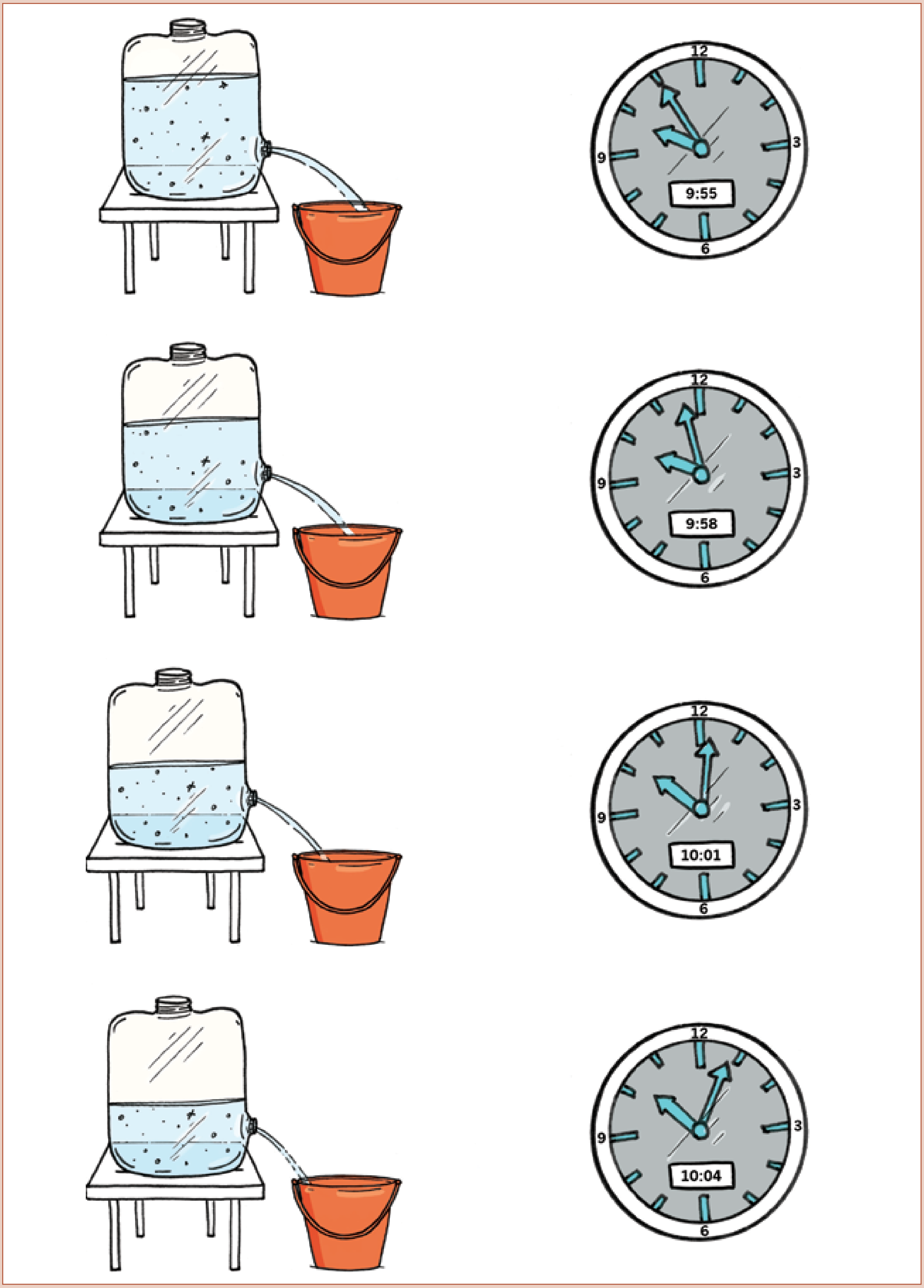
....

....

Chapter 8
Relationships between variables

In this chapter you will learn about quantities that change, such as the height of a tree. As the tree grows, the height changes. A quantity that changes is called a variable quantity or just a **variable**. It is often the case that when one quantity changes, another quantity also changes. For example, as you make more and more calls on a phone, the total cost increases. In this case, we say there is a **relationship** between the amount of money you have to pay and the number of calls you make.

You will learn how to describe a relationship between two quantities in different ways.


9:55
9:58
10:01
10:04

8 Relationships between variables

8.1 Constant and variable quantities

LOOK FOR CONNECTIONS BETWEEN QUANTITIES

1. (a) How many fingers does a person who is 14 years old have?

   ....

   (b) How many fingers does a person who is 41 years old have?

   ....

   (c) Does the number of fingers on a person's hand depend on their age? Explain.

   ....

There are two quantities in the above situation: **age** and the **number of fingers** on a person's hand. The number of fingers remains the same, irrespective of a person's age. So we say the number of fingers is a **constant** quantity. However, age changes, or varies, so we say age is a **variable** quantity.

2. Now consider each situation below. For each situation, state whether one quantity influences the other. If it does, try to say *how* the one quantity will influence the other quantity. Also say whether there is a constant in the situation.

   (a) The number of calls you make and the amount of airtime left on your cellphone

   ....

   ....

   ....

   ....

   (b) The number of houses to be built and the number of bricks required

   ....

   ....

   ....

   ....

(c) The number of learners at a school and the duration of the mathematics period

....

....

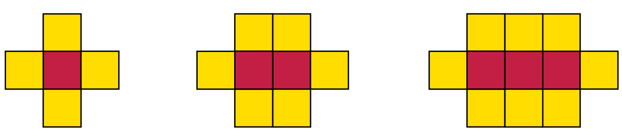
....

....

....

If one variable quantity is influenced by another, we say there is a **relationship** between the two variables. It is sometimes possible to find out what value of the one quantity, in other words what number, is linked to a specific value of the other variable.

3. Consider the following arrangements:



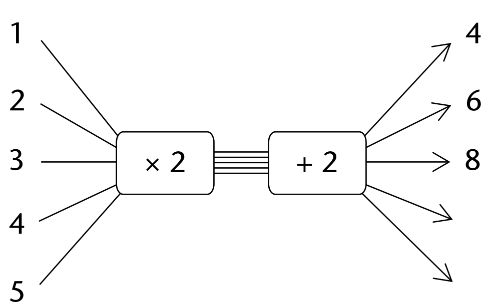
(a) How many yellow squares are there if there is only one red square? ....

(b) How many yellow squares are there if there are two red squares? ....

(c) How many yellow squares are there if there are three red squares? ....

(d) Complete the flow diagram below by filling in the missing numbers.

Can you see the connection between the arrangements above and the flow diagram? We can also describe the relationship between the red and yellow squares in words.



Input numbers
(Number of red squares)

Output numbers
(Number of yellow squares)

In words:
The number of yellow squares is found by multiplying the number of red squares by 2 and then adding 2 to the answer.

(e) How many yellow squares will there be if there are 10 red squares? ....

(f) How many yellow squares will be there if there are 21 red squares? ....

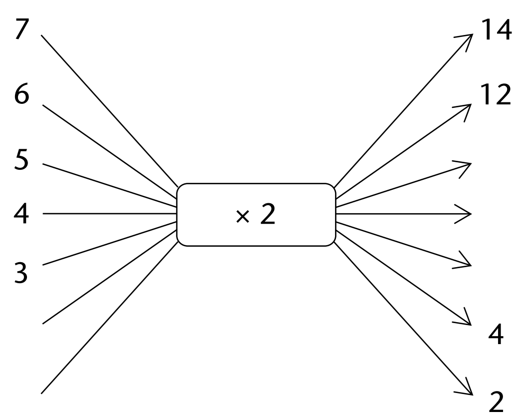
8.2 Different ways to describe relationships

COMPLETE SOME FLOW DIAGRAMS AND TABLES OF VALUES

A relationship between two quantities can be shown with a flow diagram. In a flow diagram we cannot show all the numbers, so we show only some.

1. Calculate the missing input and output numbers for the flow diagram below.

(a)



Each **input number** in a flow diagram has a corresponding **output number**. The first (top) input number corresponds to the first output number. The second input number corresponds to the second output number, and so on.
We say × 2 is the **operator**.

(b) What types of numbers are given as input numbers?

....

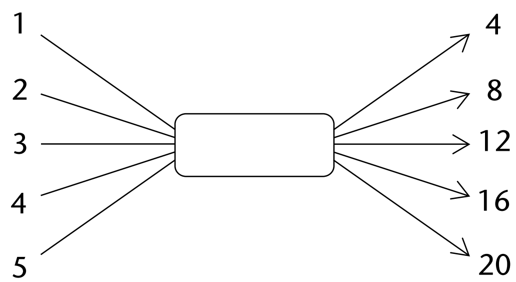
(c) In the above flow diagram, the output number 14 corresponds to the input number 7. Complete the following sentences in the same way:

In the relationship shown in the above flow diagram, the output number ...... corresponds to the input number 5.

The input number ...... corresponds to the output number 6.

If more places are added to the flow diagram, the input number ...... will correspond to the output number 40.

2. Complete this flow diagram by writing the appropriate operator, and then write the rule for completing this flow diagram in words.



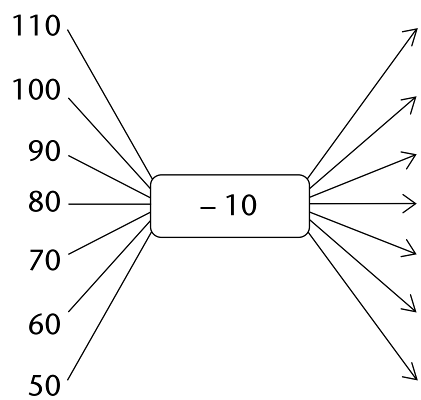
In words:

....

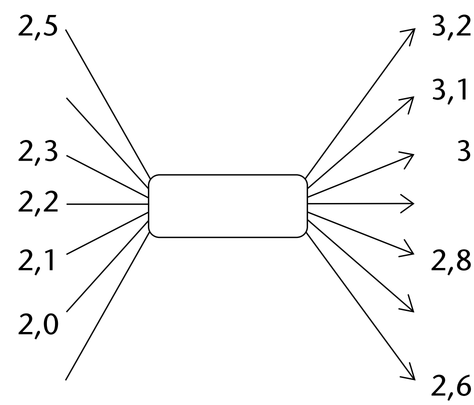
....

3. Complete the flow diagrams below. You have to find out what the operator for (b) is and fill it in yourself.

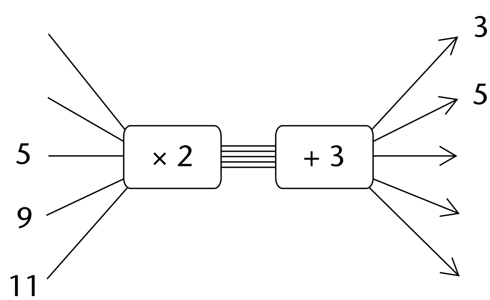
(a)



(b)



4. Complete the flow diagram:



A completed flow diagram shows two kinds of information:

- It shows what calculations are done to produce the output numbers.
- It shows which output number is connected to which input number.

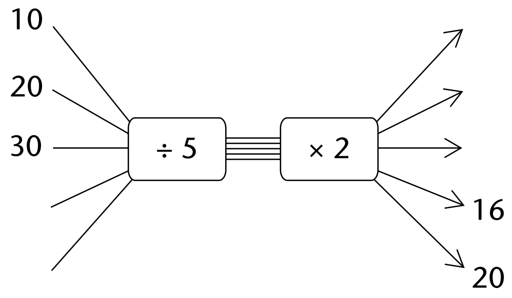
The flow diagram that you completed in question 4 shows the following information:

- Each input number is multiplied by 2 and then 3 is added to produce the output numbers.
- It shows which output number is connected to which input number.

The relationship between the input and output numbers can also be shown in a table:

Input numbers	0	1	5	9	11
Output numbers	3	5	13	21	25

5. (a) Describe in words how the output numbers below can be calculated.



....

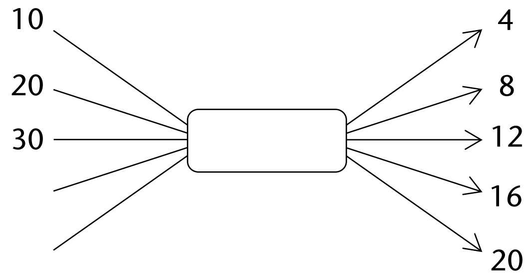
....

....

....

(b) Use the table below to show which output numbers are connected to which input numbers in the above flow diagram.

(c) Fill in the appropriate operator and complete the flow diagram.



(d) The flow diagrams in question 5(a) and 5(c) have different operators, but they produce the same output values for the same input values. Explain.

....

....

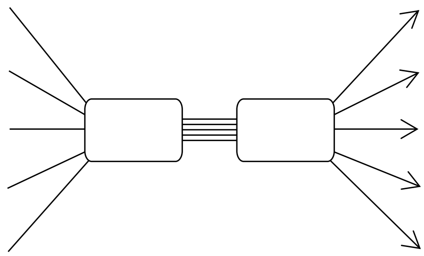
....

6. The rule for converting temperature given in degrees Celsius to degrees Fahrenheit is given as: "Multiply the degrees Celsius by 1,8 and then add 32."

(a) Check whether the table below was completed correctly. If you find a mistake, correct it.

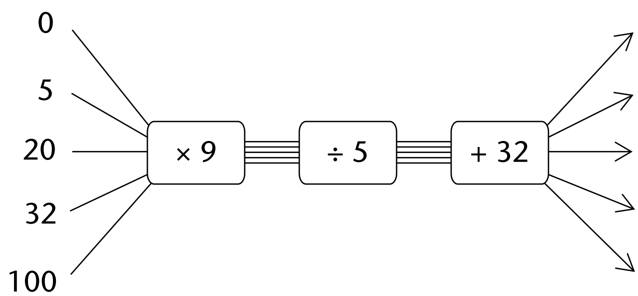
Temperature in degrees Celsius	0	5	20	32	100
Temperature in degrees Fahrenheit	32	41	68		212

(b) Complete the flow diagram to represent the information in (a).



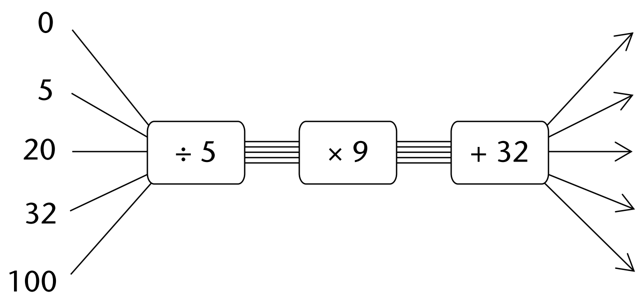
7. Another rule for converting temperature given in degrees Celsius to degrees Fahrenheit is given as: "Multiply the degrees Celsius by 9, then divide the answer by 5 and then add 32 to the answer."

(a) Complete the flow diagram below.



(b) Why do you think the flow diagrams in questions 6(b) and 7(a) produce the same output numbers for the same input numbers, even though they have different operators?

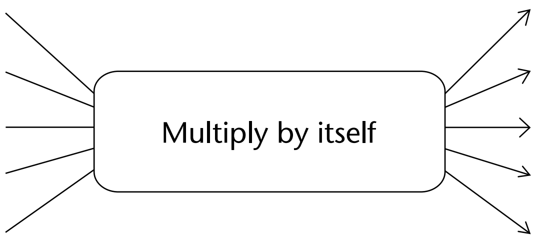
(c) Will the flow diagram below give the same output values as the flow diagram in question 7(a)? Explain.



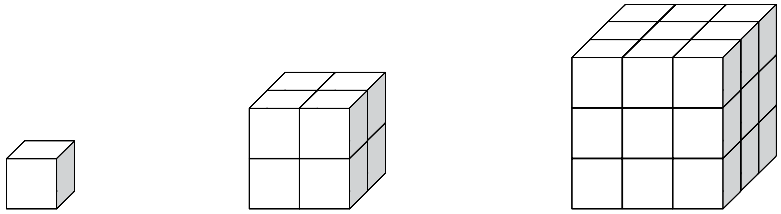
8. The rule for calculating the area of a square is: "Multiply the length of a side of the square by itself."
   (a) Complete the table below.

Length of side	4	6		10	
Area of square			64		144

   (b) Complete the flow diagram to represent the information in the table.



9. (a) The pattern below shows stacks of building blocks. The number of blocks in each stack is dependent on the number of the stack.



Stack 1 *Stack 2* *Stack 3*

Complete the table below to represent the relationship between the number of blocks and the number of the stack.

Stack number	1	2	3	4	5	6	7	8
Number of blocks	1	8						

   (b) Describe in words how the output values can be calculated.

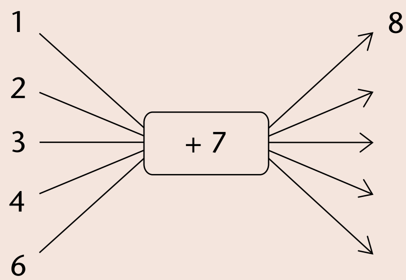
....

....

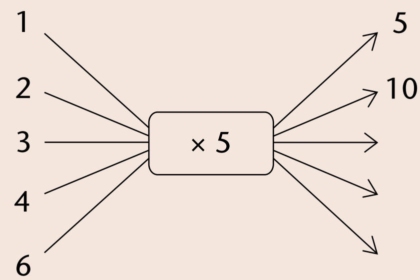
EXTENSION: LINKING FLOW DIAGRAMS, TABLES OF VALUES, AND RULES

1. Complete the flow diagrams below.

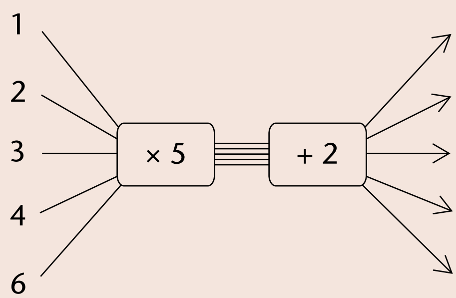
(a)



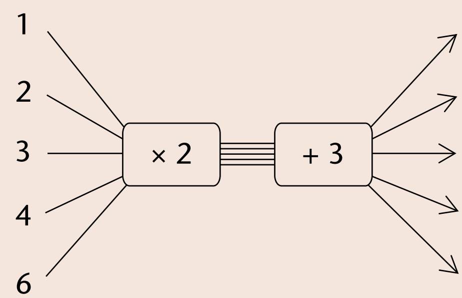
(b)



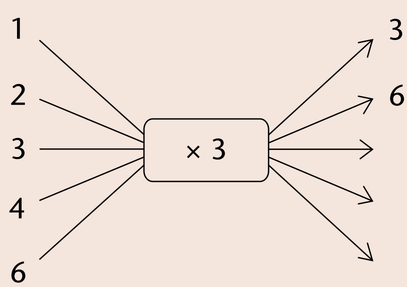
(c)



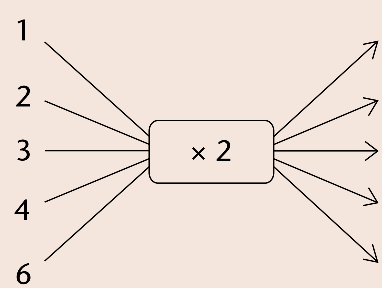
(d)



(e)



(f)



2. Calculate the differences between the consecutive output numbers and compare them to the differences between the consecutive input numbers. Consider the operator of the flow diagram. What do you notice?

....

....

3. Determine the rule to calculate the missing output numbers in this relationship and complete the table:

Input numbers	1	2	3	4	5	7	10
Output numbers	9	16	23				

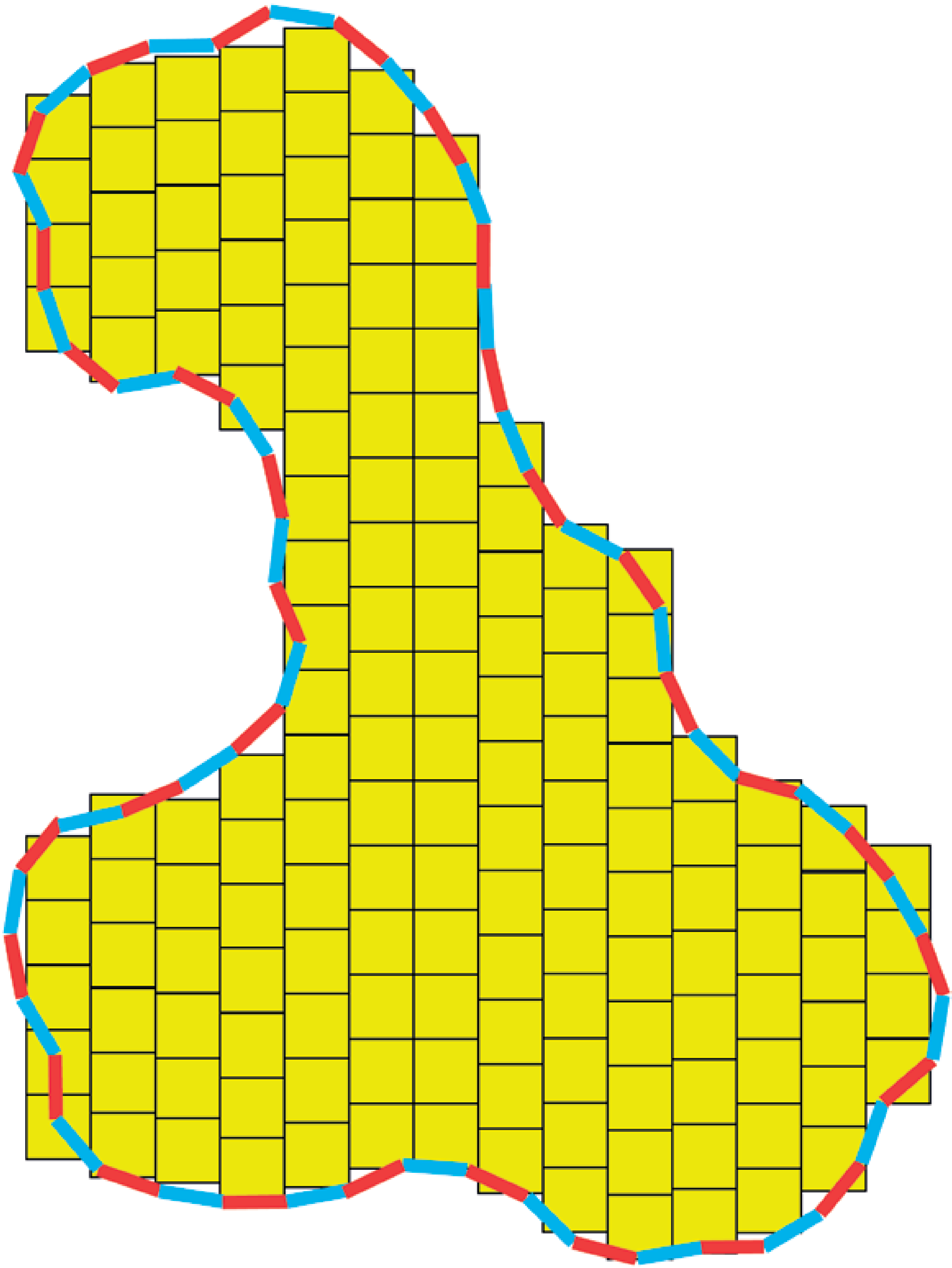
....

....

....

Chapter 9
Perimeter and area of 2D shapes

You will remember from Grade 6 that perimeter is the distance around the outermost border of something. Area is the size of a flat surface of something. In this chapter, you will learn to use different formulae to calculate the perimeter and area of squares, rectangles and triangles. You will solve problems using these formulae, and you will also learn how to convert between different units of area.



How big is it?

9 Perimeter and area of 2D shapes

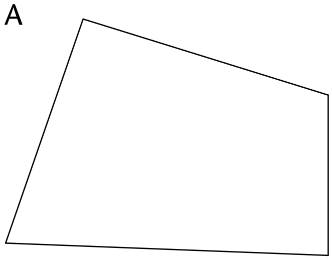
9.1 Perimeter of polygons

The **perimeter** of a shape is the total distance around the shape, or the lengths of its sides added together. Perimeter (*P*) is measured in units such as millimetres (mm), centimetres (cm) and metres (m).

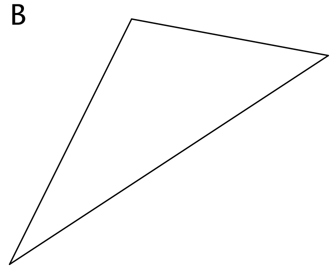
MEASURING PERIMETERS

1. (a) Use a compass and/or a ruler to measure the length of each side in figures A to C. Write the measurements in mm on each figure.
   (b) Write down the perimeter of each figure.

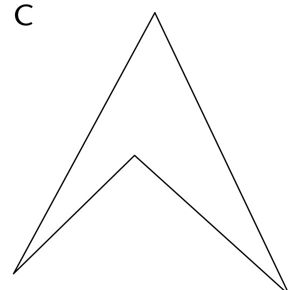
A



B



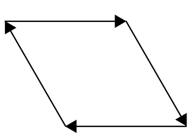
C



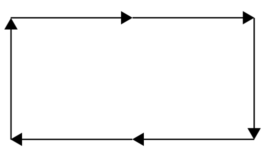
....

2. The following shapes consist of arrows that are equal in length.
   (a) What is the perimeter of each shape in number of arrows?
   (b) If each arrow is 30 mm long, what is the perimeter of each shape in mm?

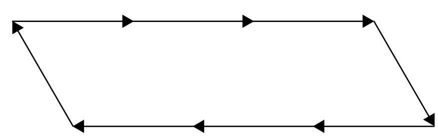
A



B

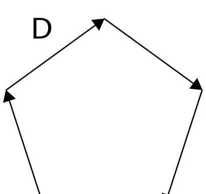


C

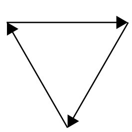


....

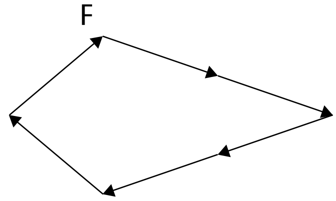
D



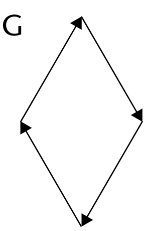
E



F



G



....

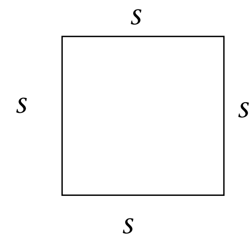
9.2 Perimeter formulae

If the sides of a square are all s units long:

Perimeter of square $= s + s + s + s$

$= 4 \times s$

or $P = 4s$



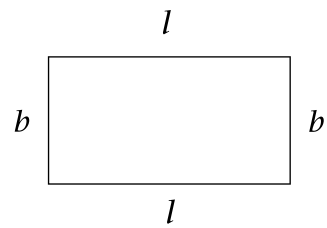
If the length of a rectangle is l units and the breadth (width) is b units:

Perimeter of rectangle $= l + l + b + b$

$= 2 \times l + 2 \times b$

$= 2l + 2b$

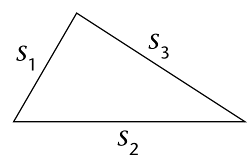
or $P = 2(l + b)$



A triangle has three sides, so:

Perimeter of triangle $= s_1 + s_2 + s_3$

or $P = s_1 + s_2 + s_3$



APPLYING PERIMETER FORMULAE

1. Calculate the perimeter of a square if the length of one of its sides is 17,5 cm.

   ....

2. One side of an equilateral triangle is 32 cm. Calculate the triangle's perimeter.

   ....

3. Calculate the length of one side of a square if the perimeter of the square is 7,2 m. (Hint: $4s = ?$ Therefore $s = ?$)

   ....

   ....

4. Two sides of a triangle are 2,5 cm each. Calculate the length of the third side if the triangle's perimeter is 6,4 cm.

   ....

   ....

5. A rectangle is 40 cm long and 25 cm wide. Calculate its perimeter.

....

....

6. Calculate the perimeter of a rectangle that is 2,4 m wide and 4 m long.

....

....

7. The perimeter of a rectangle is 8,88 m. How long is the rectangle if it is 1,2 m wide?

....

....

8. Do the necessary calculations in your exercise book in order to complete the table. (All the measurements refer to rectangles.)

	Length	Breadth	Perimeter
(a)	74 mm	30 mm	
(b)	25 mm		90 mm
(c)		1,125 cm	6,25 cm
(d)	5,5 cm		22 cm
(e)	7,5 m	3,8 m	
(f)		2,5 m	12 m

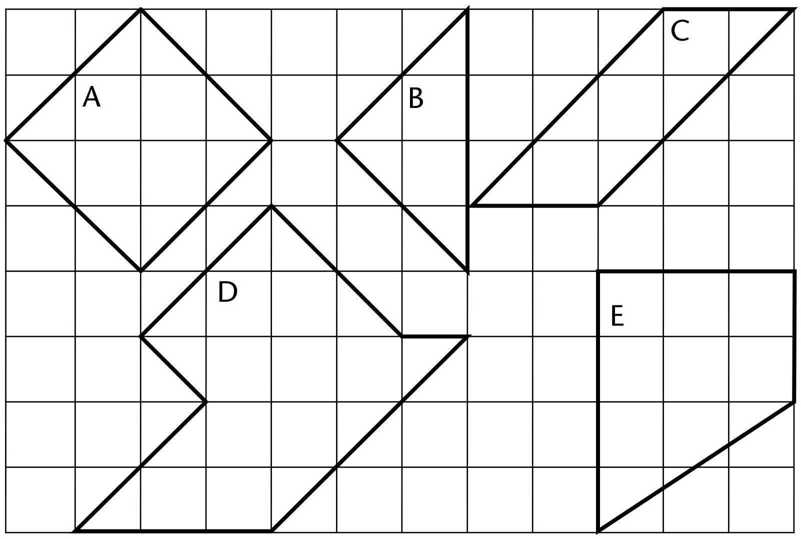
9.3 Area and square units

The **area** of a shape is the size of the flat surface surrounded by the border (perimeter) of the shape.

Usually, area (A) is measured in square units, such as square millimetres (mm^2), square centimetres (cm^2) and square metres (m^2).

SQUARE UNITS TO MEASURE AREA

1. Write down the area of figures A to E below by counting the square units. (Remember to add halves or smaller parts of squares.)



A is .... square units.

B is .... square units.

C is .... square units.

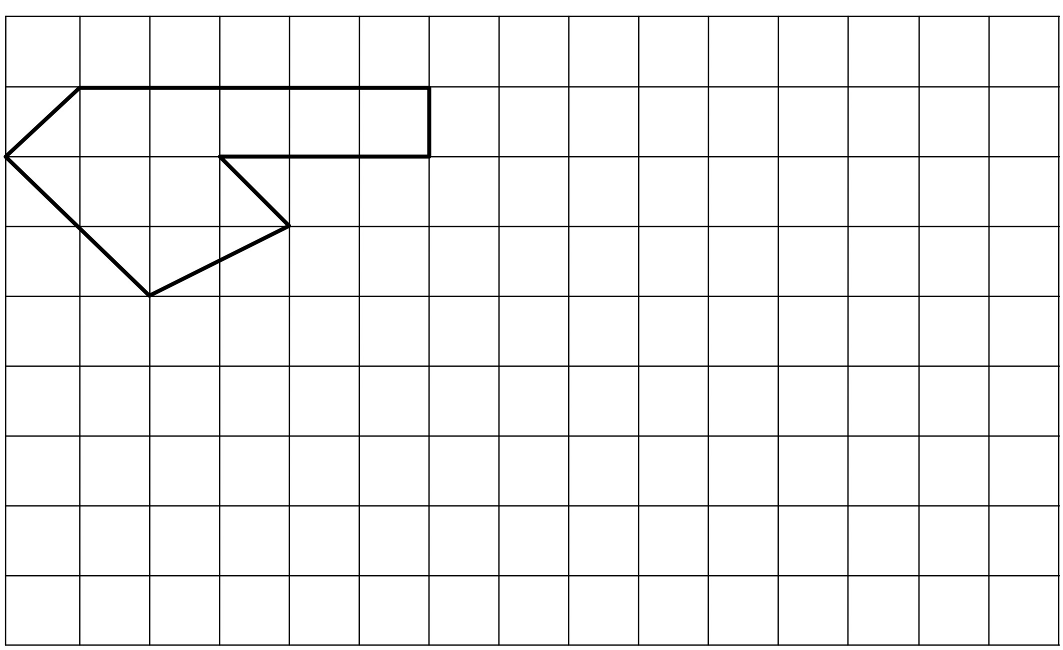
D is .... square units.

E is .... square units.

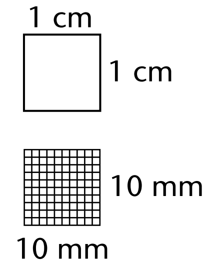
2. Each square in the grid below measures 1 cm^2 (1 cm × 1 cm).

   (a) What is the area of the shape drawn on the grid? ....

   (b) On the same grid, draw two shapes of your own. The shapes should have the same area, but different perimeters.



CONVERSION OF UNITS



The figure on the right shows a square with sides of 1 cm. The area of the square is one square centimetre (1 cm^2).

How many squares of 1 mm by 1 mm (1 mm^2) would fit into the 1 cm^2 square? ...... Complete: 1 cm^2 = ... mm^2

To change cm^2 to mm^2:

$$
\begin{aligned}
1\ cm^2 &= 1\ cm \times 1\ cm \\
&= 10\ mm \times 10\ mm \\
&= 100\ mm^2
\end{aligned}
$$

Similarly, to change mm^2 to cm^2:

$$
\begin{aligned}
1\ mm^2 &= 1\ mm \times 1\ mm \\
&= 0{,}1\ cm \times 0{,}1\ cm \\
&= 0{,}01\ cm^2
\end{aligned}
$$

We can use the same method to convert between other square units too. Complete:

From m^2 to cm^2:	**From cm^2 to m^2:**
1 m^2 = 1 m × 1 m = ... cm × .... cm = ... cm^2	1 cm^2 = 1 cm × 1 cm = 0,01 m × 0,01 m = ... m^2

So, to convert between m^2, cm^2 and mm^2 you do the following:

- cm^2 to mm^2 → multiply by 100
- m^2 to cm^2 → multiply by 10 000
- mm^2 to cm^2 → divide by 100
- cm^2 to m^2 → divide by 10 000

Do the necessary calculations in your exercise book. Then fill in your answers.

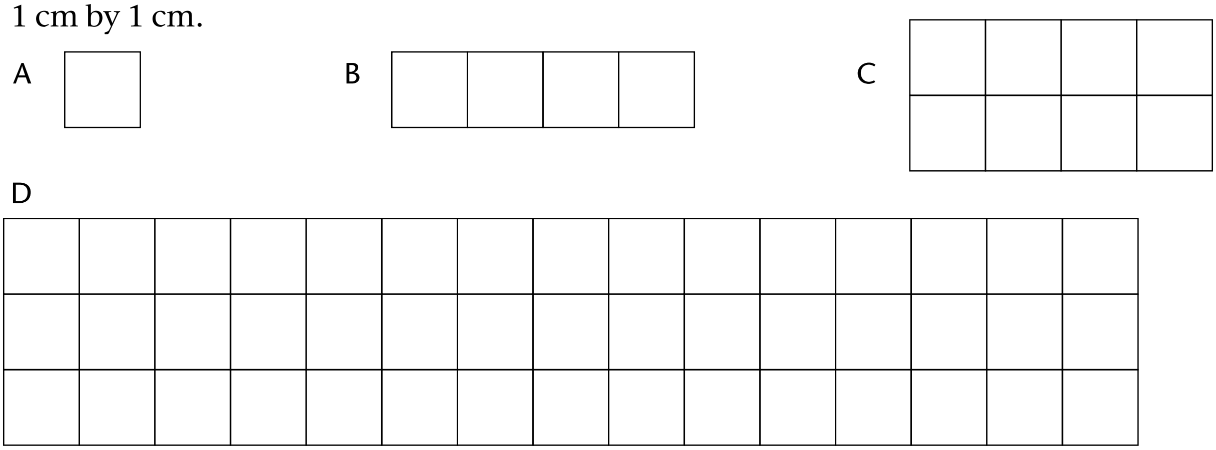
1. (a) 5 m^2 = ... cm^2 (b) 5 cm^2 = ... mm^2
   (c) 20 cm^2 = ... m^2 (d) 20 mm^2 = ... cm^2

2. (a) 25 m^2 = ... cm^2 (b) 240 000 cm^2 = ... m^2
   (c) 460,5 mm^2 = ... cm^2 (d) 0,4 m^2 = ... cm^2
   (e) 12 100 cm^2 = ... m^2 (f) 2,295 cm^2 = ... mm^2

9.4 Area of squares and rectangles

INVESTIGATING THE AREA OF SQUARES AND RECTANGLES

1. Each of the following four figures is divided into squares of equal size, namely 1 cm by 1 cm.



(a) Give the area of each figure in square centimetres (cm^2):

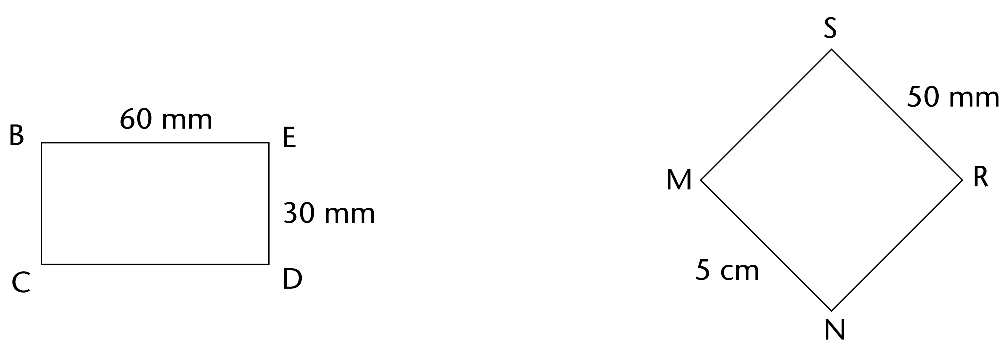
Area of A: .... Area of B: ....

Area of C: .... Area of D: ....

(b) Is there a shorter method to work out the area of each figure? Explain.

....

2. Figure BCDE is a rectangle and MNRS is a square.



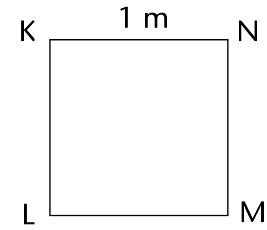
(a) How many cm^2 (1 cm × 1 cm) would fit into rectangle BCDE? ....

(b) How many mm^2 (1 mm × 1 mm) would fit into rectangle BCDE? ....

(c) What is the area of square MNRS in cm^2? ....

(d) What is the area of square MNRS in mm^2? ....

3. Figure KLMN is a square with sides of 1 m.



(a) How many squares with sides of 1 cm would fit along the length of the square? ....

(b) How many squares with sides of 1 cm would fit along the breadth of the square? ....

(c) How many squares (cm^2) would therefore fit into the whole square? ....

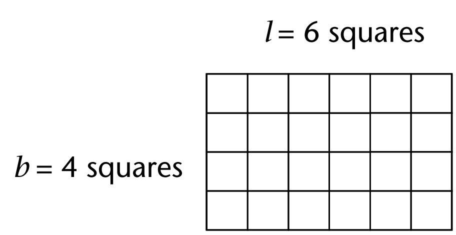
(d) Complete: 1 m^2 = ... cm^2

A quick way of calculating the number of squares that would fit into a rectangle is to multiply the number of squares that would fit along its length by the number of squares that would fit along its breadth.

FORMULAE: AREA OF RECTANGLES AND SQUARES

In the rectangle on the right:

Number of squares = Squares along the length × Squares along the breadth
= 6 × 4
= 24



From this we can deduce the following:

Area of rectangle = Length of rectangle × Breadth of rectangle
$A = l \times b$

(where A is the area in square units, l is the length and b is the breadth)

Area of square = Length of side × Length of side
$A = l \times l$
$= l^2$

(where A is the area in square units, and l is the length of a side)

The units of the values used in the calculations must be the same. Remember:

- 1 m = 100 cm and 1 cm = 10 mm
- 1 cm^2 = 1 cm × 1 cm = 10 mm × 10 mm = 100 mm^2
- 1 m^2 = 1 m × 1 m = 100 cm × 100 cm = 10 000 cm^2
- 1 mm^2 = 1 mm × 1 mm = 0,1 cm × 0,1 cm = 0,01 cm^2
- 1 cm^2 = 1 cm × 1 cm = 0,01 m × 0,01 m = 0,0001 m^2

Examples

1. Calculate the area of a rectangle with a length of 50 mm and a breadth of 3 cm. Give the answer in cm^2.

   Solution:

   Area of rectangle $= l \times b$

   $= (50 \times 30)\ \text{mm}^2$ or $A = (5 \times 3)\ \text{cm}^2$

   $= 1\,500\ \text{mm}^2$ or $= 15\ \text{cm}^2$

2. Calculate the area of a square bathroom tile with a side of 150 mm.

   Solution:

   Area of square tile $= l \times l$

   $= (150 \times 150)\ \text{mm}^2$

   $= 22\,500\ \text{mm}^2$

   The area is therefore 22 500 mm^2 (or 225 cm^2).

3. Calculate the length of a rectangle if its area is 450 cm^2 and its width is 150 mm.

   Solution:

   Area of rectangle $= l \times b$

   $450 = l \times 15$

   $30 \times 15 = l \times 15$ or $450 \div 15 = l$

   $30 = l$ $\qquad$ $30 = l$

   The length is therefore 30 cm (or 300 mm).

APPLYING THE FORMULAE

1. Calculate the area of each of the following shapes:

   (a) a rectangle with sides of 12 cm and 9 cm

   ....

   ....

   ....

   (b) a square with sides of 110 mm (answer in cm^2)

   ....

   ....

   ....

(c) a rectangle with sides of 2,5 cm and 105 mm (answer in mm^2)

....

....

....

(d) a rectangle with a length of 8 cm and a perimeter of 24 cm

....

....

....

2. A rugby field has a length of 100 m (goal post to goal post) and a breadth of 69 m.
   (a) What is the area of the field (excluding the area behind the goal posts)?

   ....

   (b) What would it cost to plant new grass on that area at a cost of R45/m^2?

   ....

   (c) Another unit for area is the hectare (ha). It is mainly used for measuring land. The size of 1 ha is the equivalent of 100 m × 100 m. Is a rugby field greater or smaller than 1 ha? Explain your answer.

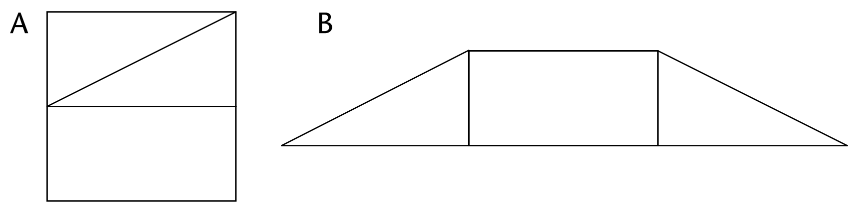
   ....

   ....

3. Do the necessary calculations in your exercise book in order to complete the table. (All the measurements refer to rectangles.)

	Length	**Breadth**	**Area**
(a)	m	8 m	120 m^2
(b)	120 mm	mm	60 cm^2
(c)	3,5 m	4,3 m	m^2
(d)	2,3 cm	cm	2,76 cm^2
(e)	5,2 m	460 cm	m^2

4. Figure A is a square with sides of 20 mm. It is cut as shown in A and the parts are combined to form figure B. Calculate the area of figure B.

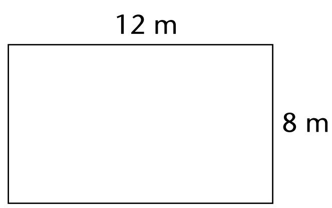


....

....

....

5. Margie plants a vegetable patch measuring 12 m × 8 m.



(a) What is the area of the vegetable patch?

....

....

....

(b) She plants carrots on half of the patch, and tomatoes and potatoes on a quarter of the patch each. Calculate the area covered by each type of vegetable?

....

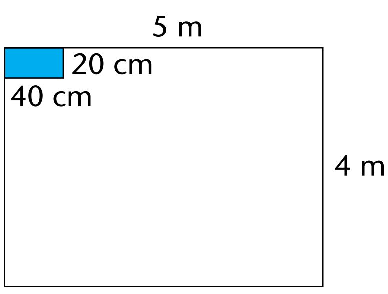
....

(c) How much will she pay to put fencing around the patch? The fencing costs R38/m.

....

....

6. Mr Allie has to tile a kitchen floor measuring 5 m × 4 m. The blue tiles he uses each measure 40 cm × 20 cm.



(a) How many tiles does Mr Allie need?

....

....

....

....

(b) The tiles are sold in boxes containing 20 tiles. How many boxes should he buy?

....

....

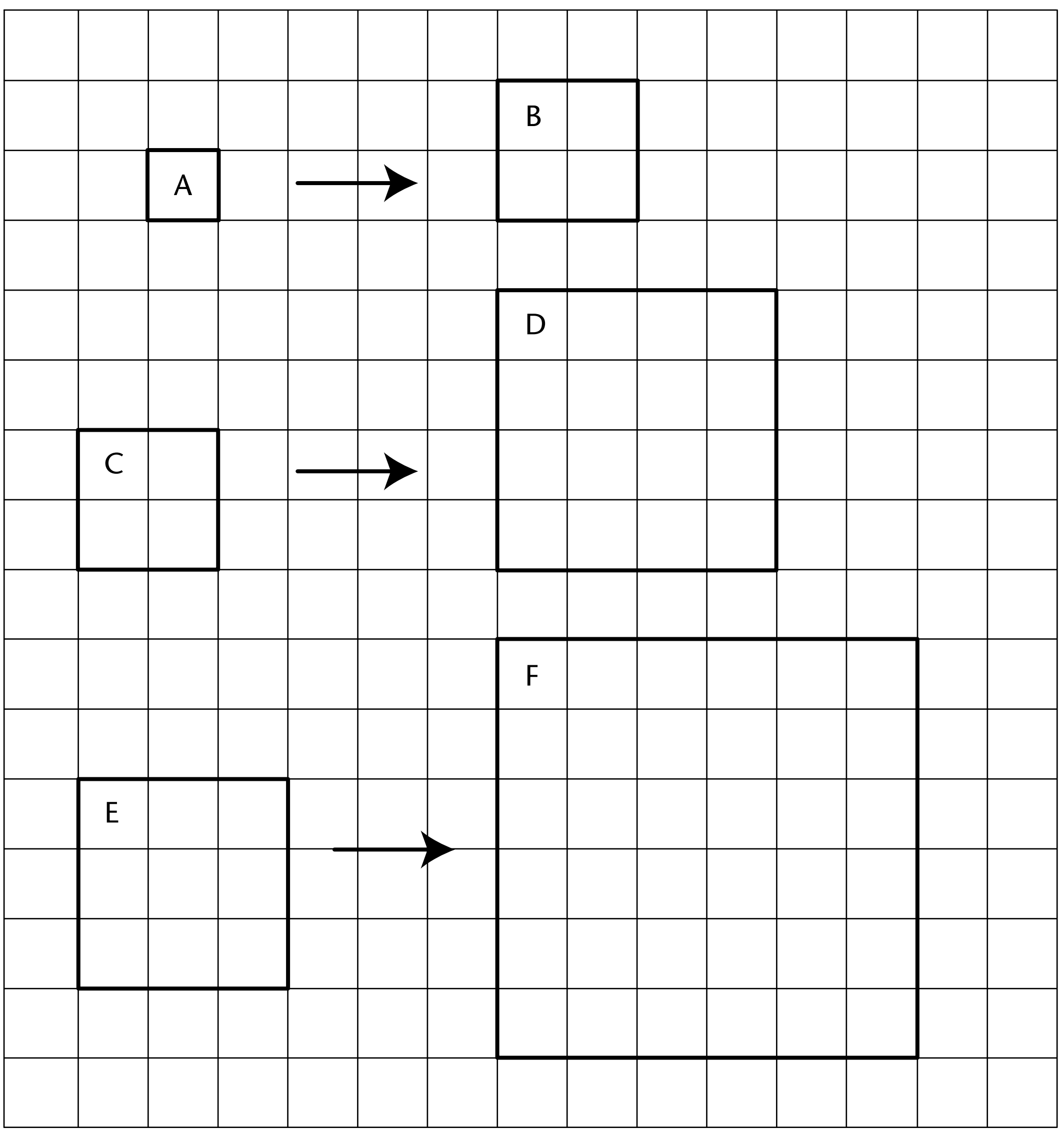
DOUBLING A SIDE AND ITS EFFECT ON AREA

When a side of a square is doubled, will the area of the square also be doubled?

The size of each square making up the grid below is 1 cm × 1 cm.

1. (a) For each square drawn on the grid, label the lengths of its sides.
   (b) Write down the area of each square. (Write the answer inside the square.)
2. Notice that the second square in each pair of squares has a side length that is double the side length of the first square.
3. Compare the areas of the squares in each pair; then complete the following:
   When the side of a square is doubled, its area

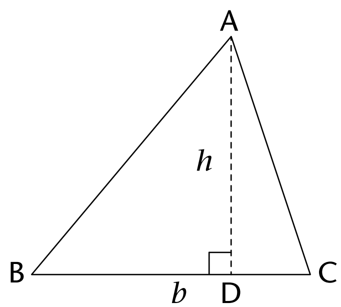
   ....



9.5 Area of triangles

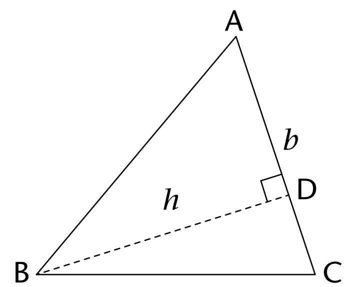
HEIGHTS AND BASES OF A TRIANGLE

The **height (*h*)** of a triangle is a perpendicular line segment drawn from a vertex to its opposite side. The opposite side, which forms a right angle with the height, is called the **base (*b*)** of the triangle. Any triangle has three heights and three bases.



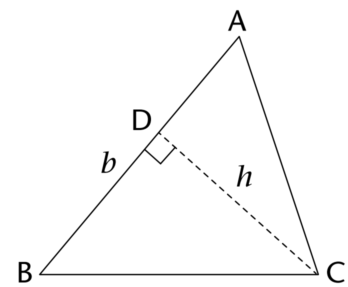
AD = height

BC = base



BD = height

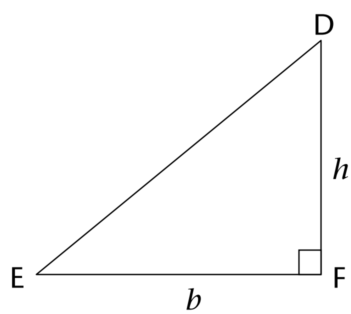
AC = base



CD = height

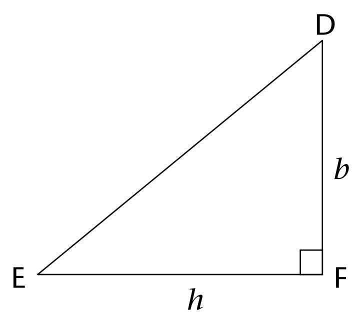
AB = base

In a right-angled triangle, two sides are already at right angles:



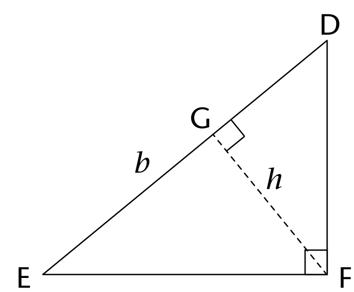
DF = height

EF = base



EF = height

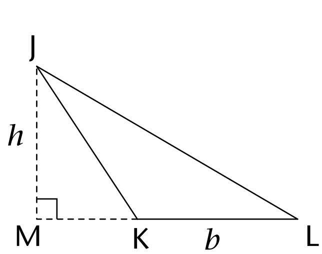
DF = base



FG = height

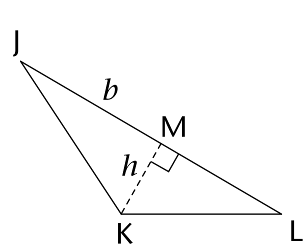
DE = base

Sometimes a base must be extended outside of the triangle in order to draw the perpendicular height. This is shown in the first and third triangles below. Note that the extended part does not form part of the base's measurement:



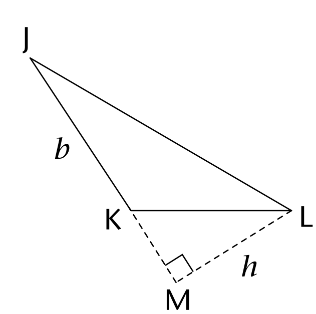
JM = height

KL = base



KM = height

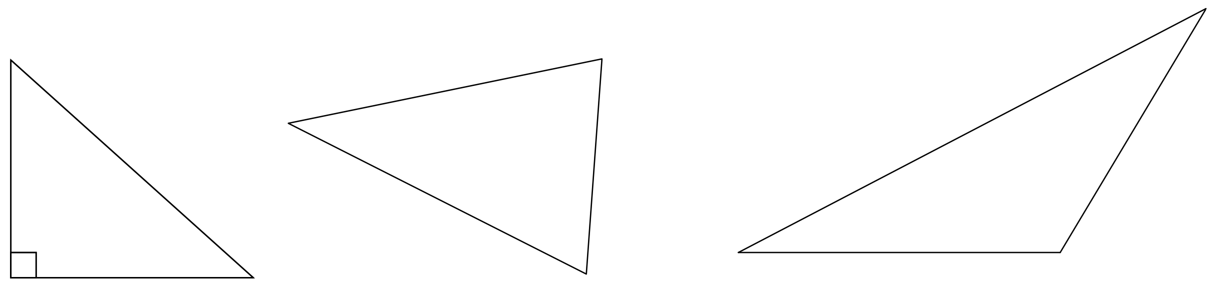
JL = base



LM = height

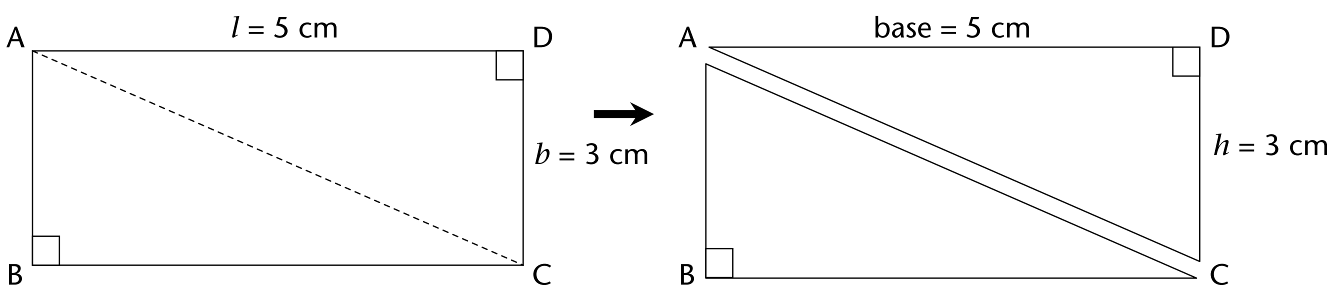
JK = base

1. Draw any height in each of the following triangles. Label the height (h) and base (b) on each triangle.
2. Label another set of heights and bases on each triangle.



FORMULA: AREA OF A TRIANGLE

ABCD is a rectangle with length = 5 cm and breadth = 3 cm. When A and C are joined, it creates two triangles that are equal in area: ΔABC and ΔADC.



$$\text{Area of rectangle} = l \times b$$

$$\text{Area of } \Delta ABC \text{ (or } \Delta ADC) = \frac{1}{2} \text{(Area of rectangle)}$$

$$= \frac{1}{2} (l \times b)$$

In rectangle ABCD, AD is its length and CD is its breadth.
But look at ΔADC. Can you see that AD is a base and CD is its height?

So instead of saying:

$$\text{Area of } \Delta ADC \text{ or any other triangle} = \frac{1}{2} (l \times b)$$

we say:

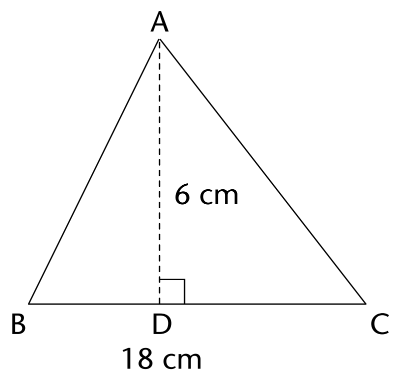
$$\textbf{Area of a triangle} = \frac{1}{2} \text{(base × height)}$$

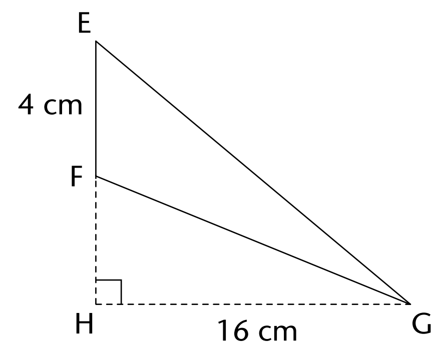
$$= \frac{1}{2} (b \times h)$$

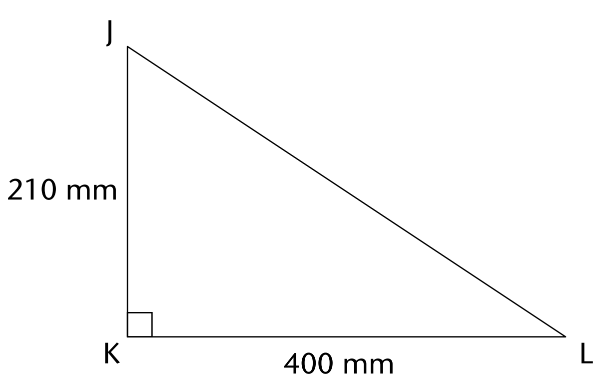
In the formula for the area of a triangle, b means 'base' and not 'breadth', and h means perpendicular height.

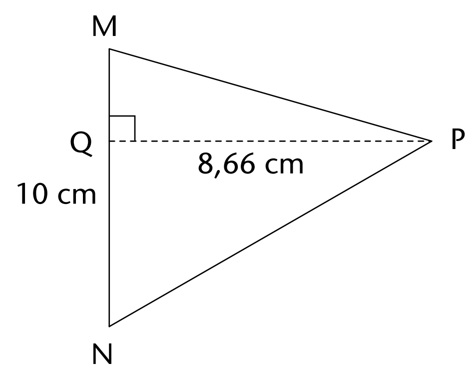
APPLYING THE AREA FORMULA

1. Use the formula to calculate the areas of the following triangles: ΔABC, ΔEFG, ΔJKL and ΔMNP.



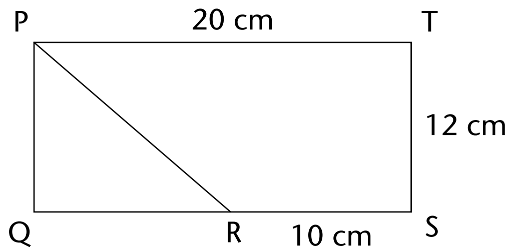




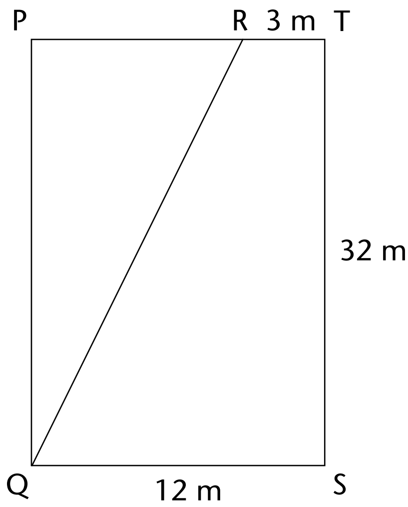


2. PQST is a rectangle in each case below. Calculate the area of ΔPQR each time.

(a)



(b)



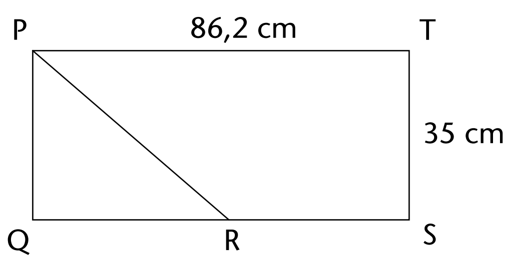
. . . .

. . . .

. . . .

. . . .

(c) R is the midpoint of QS.



. . . .

. . . .

3. In ΔABC, the area is 42 m^2, and the perpendicular height is 16 m. Find the length of the base.

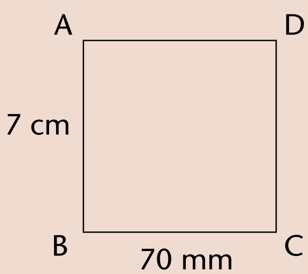
. . . .

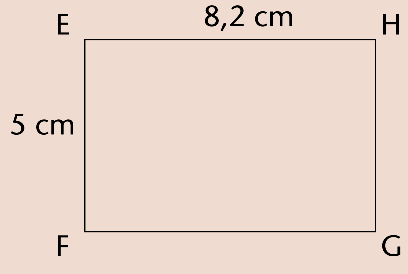
. . . .

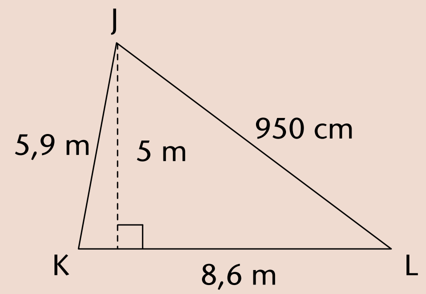
. . . .

WORKSHEET

1. Calculate the perimeter (P) and area (A) of the following figures:





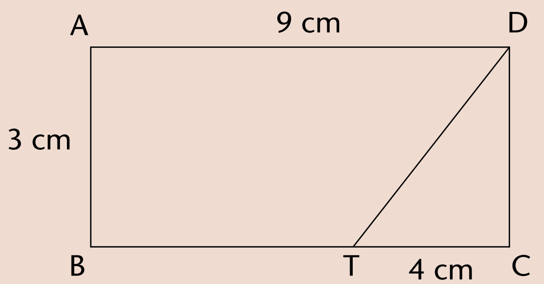


$P =$ .... $P =$ .... $P =$ ....

$A =$ .... $A =$ .... $A =$ ....

....

2. Figure ABCD is a rectangle:
   AB = 3 cm, AD = 9 cm and TC = 4 cm.



(a) Calculate the perimeter of ABCD.

....

....

....

(b) Calculate the area of ABCD.

....

....

....

(c) Calculate the area of ΔDTC.

....

....

....

(d) Calculate the area of ABTD.

....

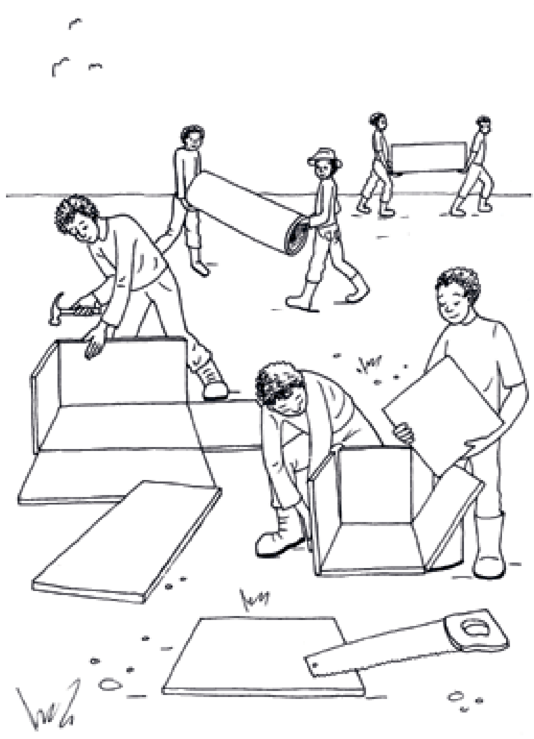
....

....

CHAPTER 10

Surface area and volume of 3D objects

In this chapter, you will investigate the formulae we can use to calculate the area of the outer surfaces of cubes and rectangular prisms. Using nets of these 3D objects will help you to understand how we get to these formulae. You will then explore the formulae we can use to calculate the amount of space that solid cubes and rectangular prisms take up. The amount of space is known as their volume. You will then come to understand the difference between the volume and the capacity of cubes and rectangular prisms. You will also learn about the units that are used to calculate surface area, volume and capacity, and you will find out how to convert between different units of measurement.



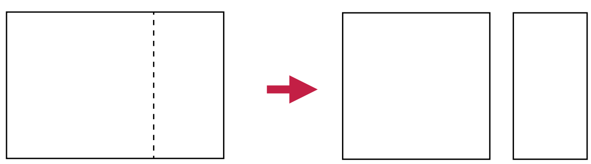
10 Surface area and volume of 3D objects

10.1 Surface area of cubes and rectangular prisms

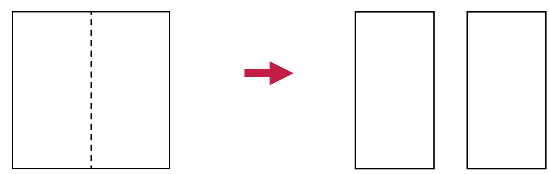
INVESTIGATING SURFACE AREA

1. Follow the instructions below to make a paper cube.

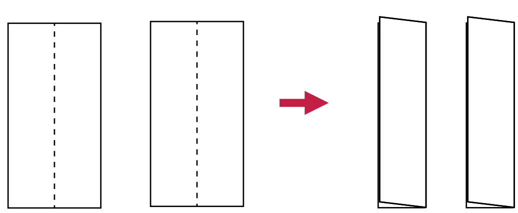
   Step 1: Cut off part of an A4 sheet so that you are left with a square.

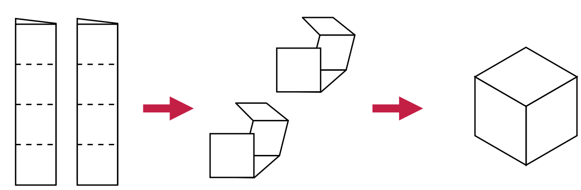
   Step 2: Cut the square into two equal halves.

   Step 3: Fold each half square lengthwise down the middle to form two double-layered strips.

   Step 4: Fold each strip into four square sections, and put the two parts together to form a paper cube. Use sticky tape to keep it together.

2. Number each face of the cube. How many faces does the cube have? ....
3. Measure the side length of one face of the cube. ....
4. Calculate the area of one face of the cube. ....
5. Add up the areas of all the faces of the cube.

   ....

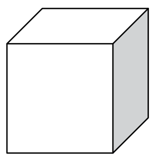
The **surface area** of an object is the sum of the areas of all its faces (or outer surfaces).

As for other areas, we measure surface area in square units, for example mm^2, cm^2, m^2.

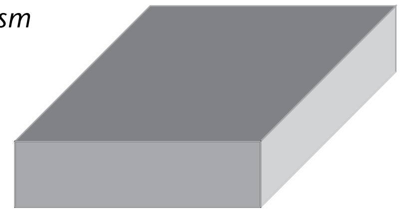
A cube has six identical square faces. A die (plural: dice) is an example of a cube.

A rectangular prism also has six faces, but its faces can be squares and/or rectangles. A matchbox is an example of a rectangular prism.

Cube



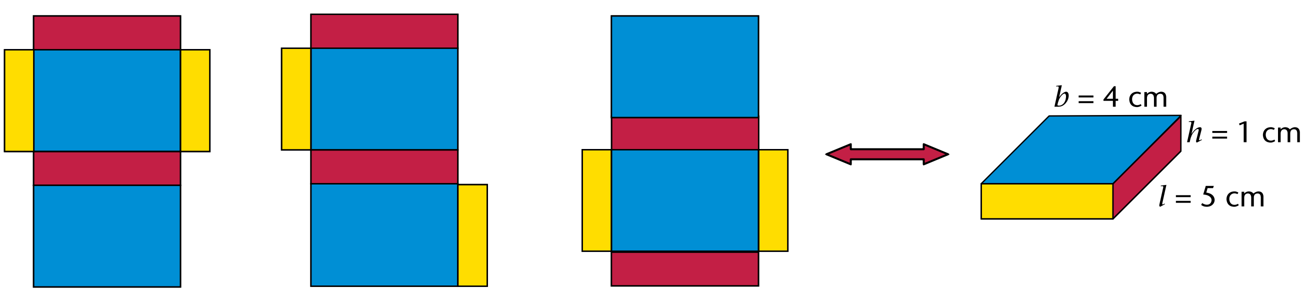
Rectangular prism

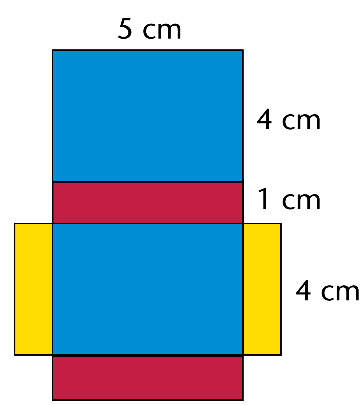


USING NETS OF RECTANGULAR PRISMS AND CUBES

It is sometimes easier to see all the faces of a rectangular prism or cube if we look at its net. A **net** of a prism is the figure obtained when cutting the prism along some of its edges, unfolding it and laying it flat.

1. Take a sheet of paper and wrap it around a matchbox so that it covers the whole box without going over the same place twice. Cut off extra bits of paper as necessary so that you have only the paper that covers each face of the matchbox.
2. Flatten the paper and draw lines where the paper has been folded. Your sheet might look like one of the following nets (there are also other possibilities):





3. Notice that there are six rectangles in the net, each matching a rectangular face of the matchbox. Point to the three pairs of identical rectangles in each net.
4. Use the measurements given to work out the surface area of the prism. (Add up the areas of each face.)

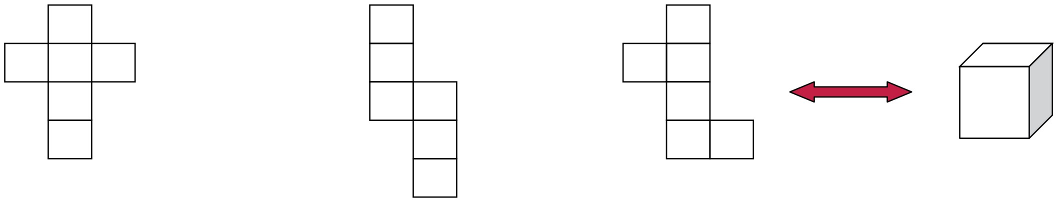
   ....

   ....

5. Explain to a classmate why you think the following formula is or is not correct:

   Surface area of a rectangular prism = $2(l \times b) + 2(l \times h) + 2(b \times h)$

6. Here are three different nets of the same cube.



(a) Can you picture in your mind how the squares can fold up to make a cube?

(b) If the length of an edge of the cube is 1 cm, what is the area of one of its faces? .....

What then is the area of all its six faces? ....

(c) Explain to a classmate why you think the following formula is or is not correct:
Surface area of a cube $= 6(l \times l) = 6l^2$

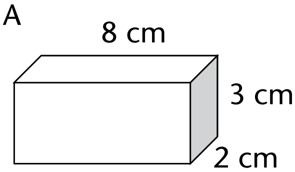
(d) If the length of an edge of the cube above is 3 cm, what is the surface area of the cube?

....

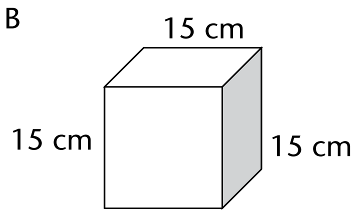
WORKING OUT SURFACE AREAS

1. Work out the surface areas of the following rectangular prisms and cubes.

A



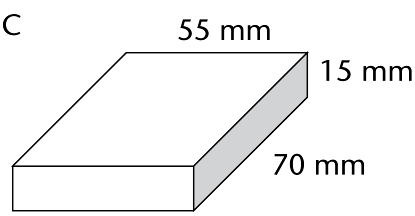
B



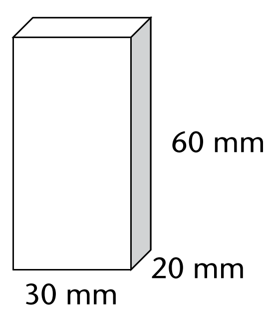
....

....

C



D

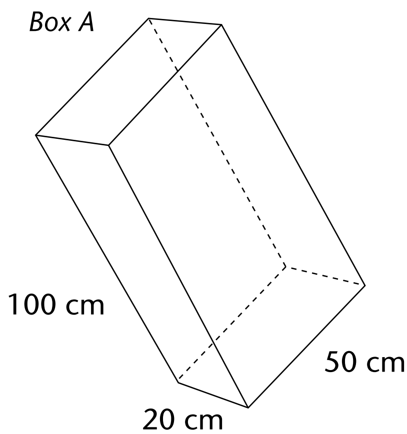


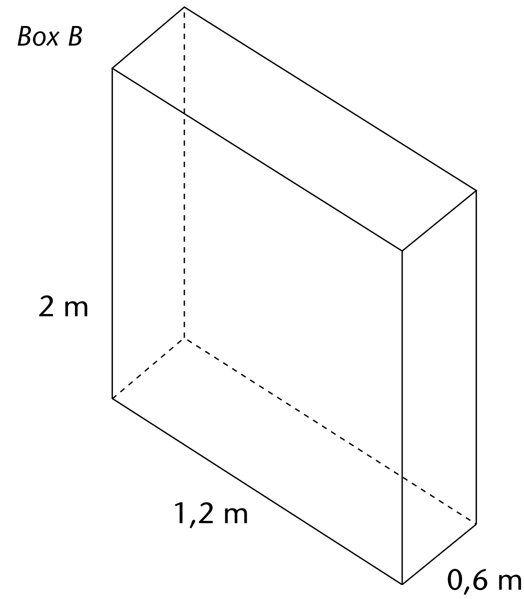
....

....

....

2. The following two boxes are rectangular prisms. The boxes must be painted.





(a) Calculate the total surface area of box A and of box B.

....

....

....

....

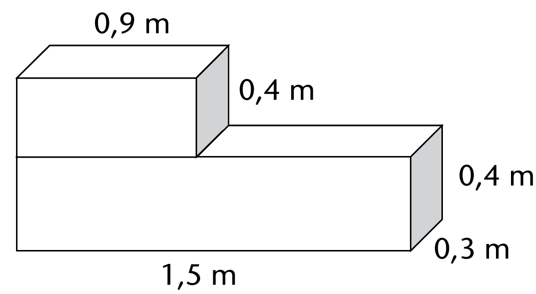
(b) What will it cost to paint both boxes if the paint costs R1,34 per m^2?

....

....

....

3. Two cartons, which are rectangular prisms, are glued together as shown. Calculate the surface area of this object. (Note which faces can be seen and which cannot.)



....

....

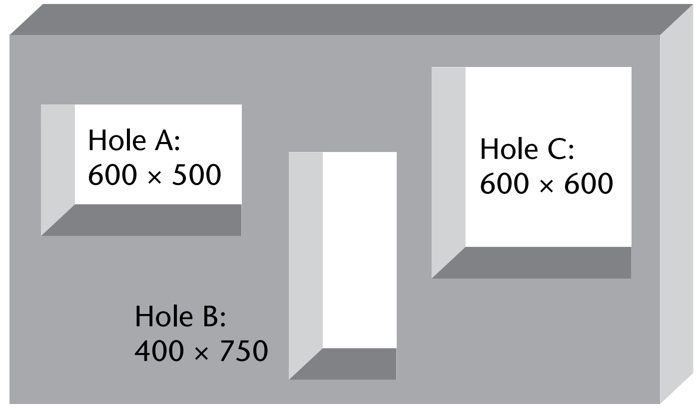
....

....

....

....

4. This large plastic wall measures 3 m × 0,5 m × 1,5 m. It has to be painted for the Uyavula Literacy Project. The wall has three holes in it, labelled A, B and C, as shown. The holes go right through the wall. The measurements of the holes are in mm.



(a) Calculate the area of the front and back surfaces that must be painted.

Remember from the previous chapter:
$1\ cm^2 = 100\ mm^2$
$1\ m^2 = 10\ 000\ cm^2$

....

....

....

....

....

....

....

(b) Calculate the area of the two side faces, as well as the top face.

....

....

....

(c) Calculate the total surface area of the wall, excluding the bottom and the inner surfaces where the holes are, because these will not be painted.

....

....

....

(d) What will it cost if the water-based paint costs R2,00 per m^2?

....

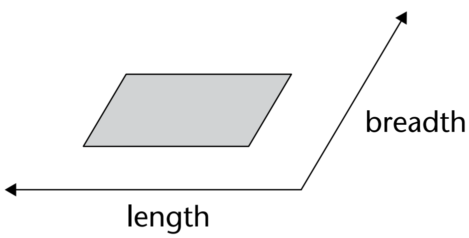
....

....

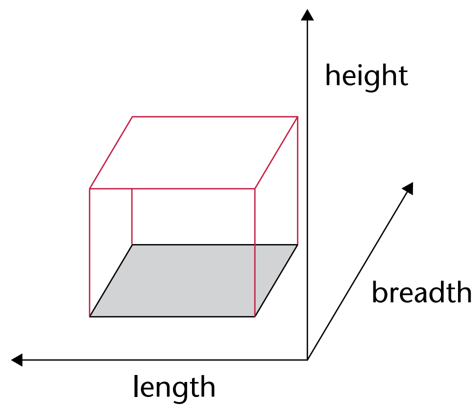
10.2 Volume of rectangular prisms and cubes

2D shapes are flat and have only two dimensions, namely length (l) and breadth (b). 3D objects have three dimensions, namely length (l), breadth (b) and height (h). You can think of a dimension as a direction in space. Look at these examples:

2D shape: rectangle



3D object: rectangular prism



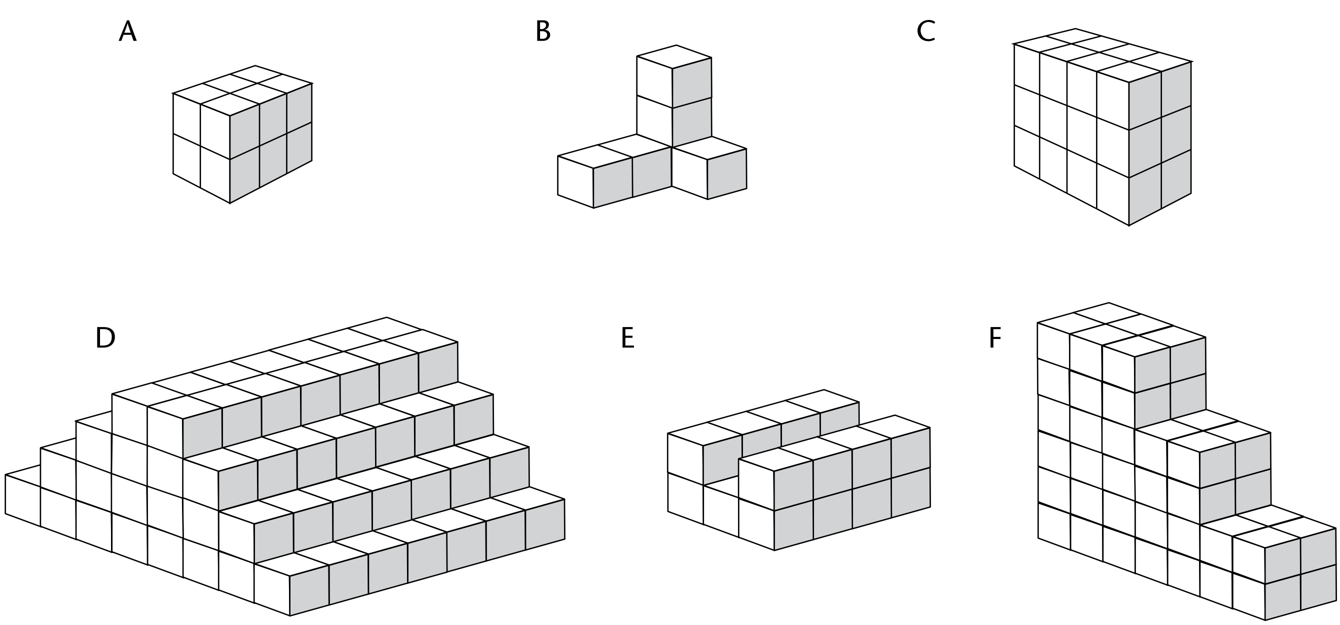
3D objects therefore take up space in a way that 2D shapes do not. We can measure the amount of space that 3D objects take up.

Every object in the real world is 3D. Even a sheet of paper is a 3D object. Its height is about 0,1 mm.

CUBES TO MEASURE AMOUNT OF SPACE

We can use cubes to measure the amount of space that an object takes up.

1. Identical toy building cubes were used to make the stacks shown below.



(a) Which stack takes up the least space? ....

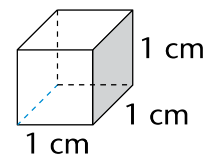
(b) Which stack takes up the most space? ....

(c) Order the stacks from the one that takes up the least space to the one that takes up the most space. (Write the letters of the stacks.)

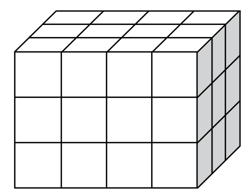
The space (in all directions) occupied by a 3D object is called its **volume**.

Cubes are the units we use to measure volume.

A cube with edges of 1 cm (that is, 1 cm × 1 cm × 1 cm) has a volume of one cubic centimetre (1 cm^3).

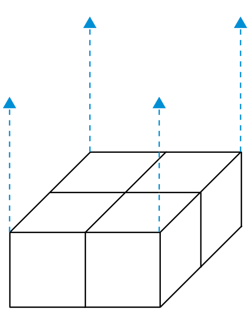


2. The figure on the right shows a rectangular prism made from 36 cubes, each with an edge length of 1 cm. The prism thus has a volume of 36 cubic centimetres (36 cm^3).

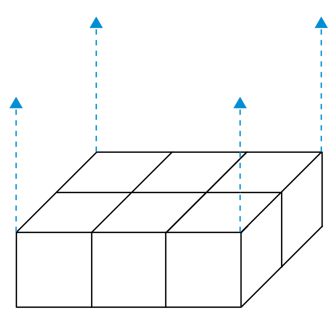


(a) The stack is taken apart and all 36 cubes are stacked again to make a new rectangular prism with a base of four cubes (see A below.) How many layers of cubes will the new prism be? What is the height of the new prism?

A



B



(b) Repeat (a), but this time make a prism with a base of six cubes (see B above).

(c) Which one of the rectangular prisms in questions (a) and (b) takes up the most space in all directions? (Which one has the greatest volume?)

(d) What will be the volume of the prism in question (b) if there are 7 layers of cubes altogether?

....

....

(e) A prism is built with 48 cubes, each with an edge length of 1 cm. The base consists of 8 layers. What is the height of the prism?

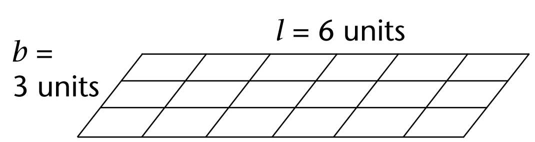
....

....

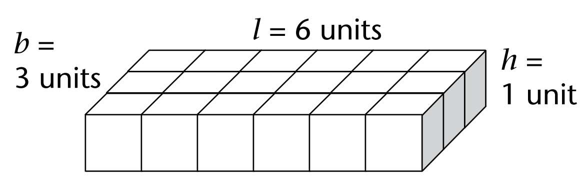
FORMULA TO CALCULATE VOLUME

You can think about the volume of a rectangular prism in the following way:

Step 1: Measure the area of the bottom face (also called the base) of a rectangular prism. For the prism given here: $A = l \times b = 6 \times 3 = 18$ square units.



Step 2: A layer of cubes, each 1 unit high, is placed on the flat base. The base now holds 18 cubes. It is $6 \times 3 \times 1$ cubic units.

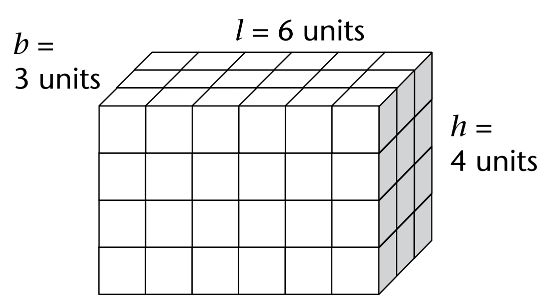


Step 3: Three more layers of cubes are added so that there are 4 layers altogether. The prism's height (h) is 4 units. The volume of the prism is:

$V = (6 \times 3) \times 4$

or V = Area of base × number of layers

$= (l \times b) \times h$



Therefore:

Volume of a rectangular prism = Area of base × height

$= l \times b \times h$

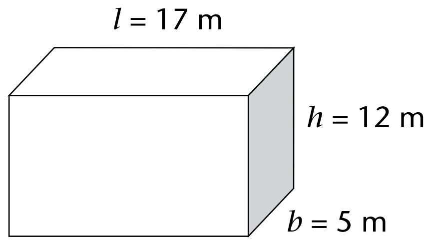
Volume of a cube $= l \times l \times l$ (edges are all the same length)

$= l^3$

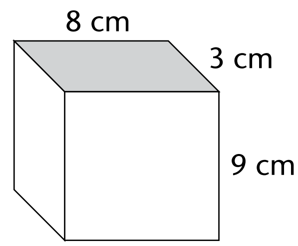
APPLYING THE FORMULAE

1. Calculate the volume of these prisms and cubes.

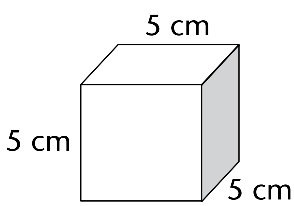
A



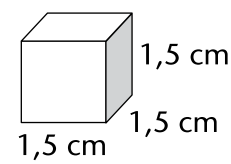
B



C



D



2. Calculate the volume of prisms with the following measurements:

(a) $l = 7$ m, $b = 6$ m, $h = 6$ m

(b) $l = 55$ cm, $b = 10$ cm, $h = 20$ cm

(c) Surface of base = 48 m^2, $h = 4$ m

(d) Surface of base = 16 mm^2, $h = 12$ mm

3. Calculate the volume of cubes with the following edge lengths:

(a) 7 cm

(b) 12 mm

4. Calculate the volume of the following square-based prisms:
   (a) side of the base = 5 mm, h = 12 mm   (b) side of the base = 11 m, h = 800 cm

   ....

   ....

   ....

   ....

5. The volume of a prism is 375 m^3. What is the height of the prism if its length is 8 m and its breadth is 15 m?

   ....

   ....

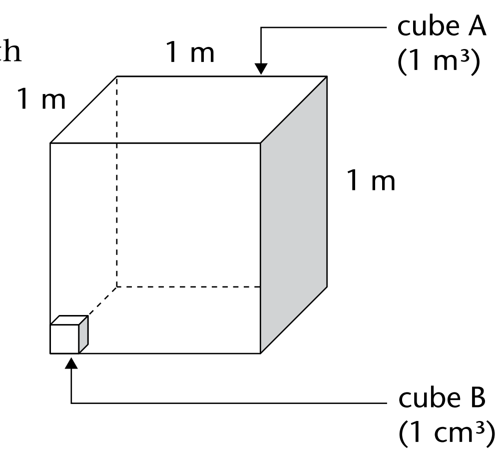
   ....

   ....

10.3 Converting between cubic units

CUBIC UNITS TO MEASURE VOLUME

This drawing shows a cube (A) with an edge length of 1 m. Also shown is a small cube (B) with an edge length of 1 cm.



How many small cubes can fit inside the large cube?

- 100 small cubes can fit along the length of the base of cube A (because there are 100 cm in 1 m).
- 100 small cubes can fit along the breadth of the base of cube A.
- 100 small cubes can fit along the height of cube A.

$$
\begin{aligned}
\text{Total number of 1 cm}^3 \text{ cubes in 1 m}^3 &= 100 \times 100 \times 100 \\
&= 1\,000\,000 \\
\therefore \quad 1 \text{ m}^3 &= 1\,000\,000 \text{ cm}^3
\end{aligned}
$$

Work out how many mm^3 are equal to 1 cm^3:

$$\begin{aligned} 1\text{ cm}^3 &= 1\text{ cm} \times 1\text{ cm} \times 1\text{ cm} \\ &= \ldots\ldots\text{mm} \times \ldots\ldots\text{mm} \times \ldots\ldots\text{mm} \\ &= \ldots\ldots\ldots\text{mm}^3 \end{aligned}$$

Cubic units:

1 m^3 = 1 000 000 cm^3
(multiply by 1 000 000 to change m^3 to cm^3)

1 cm^3 = 0,000001 m^3
(divide by 1 000 000 to change cm^3 to m^3)

1 cm^3 = 1 000 mm^3
(multiply by 1 000 to change cm^3 to mm^3)

1 mm^3 = 0,001 cm^3
(divide by 1 000 to change mm^3 to cm^3)

WORKING WITH CUBIC UNITS

1. Which unit, the cubic centimetre (cm^3) or the cubic metre (m^3), would be used to measure the volume of each of the following?

   (a) a bar of soap ....
   (b) a book ....
   (c) a wooden rafter for a roof ....
   (d) sand on a truck ....
   (e) a rectangular concrete wall ....
   (f) a die ....
   (g) water in a swimming pool ....
   (h) medicine in a syringe ....

2. Write the following volumes in cm^3:

   (a) 1 000 mm^3 ....
   (b) 3 000 mm^3 ....
   (c) 2 500 mm^3 ....
   (d) 4 450 mm^3 ....
   (e) 7 824 mm^3 ....
   (f) 50 mm^3 ....

3. Write the following volumes in m^3:

   (a) 1 000 000 cm^3 ....
   (b) 4 000 000 cm^3 ....
   (c) 1 500 000 cm^3 ....
   (d) 2 350 000 cm^3 ....
   (e) 500 000 cm^3 ....
   (f) 350 000 cm^3 ....

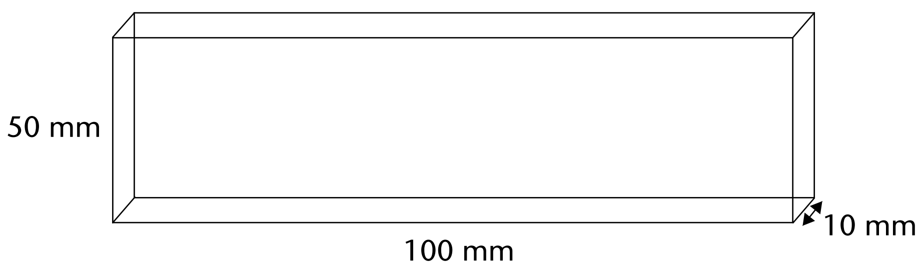
4. Write the following volumes in cm^3:

(a) 2 000 mm^3 ....
(b) 4 120 mm^3 ....
(c) 1,5 m^3 ....
(d) 34 m^3 ....
(e) 50 000 mm^3 ....
(f) 2,23 m^3 ....

5. A rectangular hole has been dug for a children's swimming pool. It is 7 m long, 4 m wide and 1 m deep. What is the volume of earth that has been dug out?

....

6. Calculate the volume of wood in the plank shown below. Answer in cm^3.

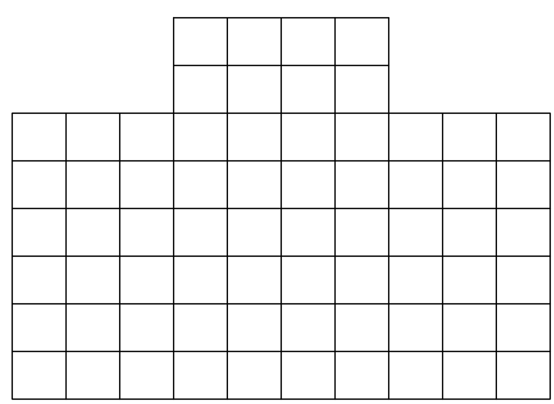


....

....

....

7. The drawing shows the base (viewed from below) of a stack built with 1 cm^3 cubes. The stack is 80 mm high everywhere.



(a) What is the volume of the stack?

....

....

(b) Complete the following:
Volume of stack = area of base ....

8. Calculate the volume of each of the following rectangular prisms:
   (a) length = 20 cm; breadth = 15 cm; height = 10 cm

   ....

   ....

   ....

   (b) length = 130 mm; breadth = 10 cm; height = 5 mm

   ....

   ....

   ....

   (c) length = 1 200 cm; breadth = 5,5 m; height = 3 m

   ....

   ....

   ....

   (d) length = 1,2 m; breadth = 2,25 m; height = 4 m

   ....

   ....

   ....

   (e) area of base = 300 cm^2; height = 150 mm

   ....

   ....

   ....

   (f) area of base = 12 m^2; height = 2,25 m

   ....

   ....

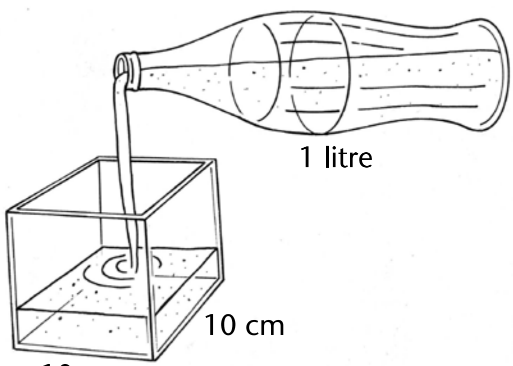
   ....

10.4 Volume and capacity

The space inside a container is called the internal volume, or **capacity**, of the container. Capacity is often measured in units of millilitres (ml), litres (ℓ) and kilolitres (kl). However, it can also be measured in cubic units.

EQUIVALENT UNITS FOR VOLUME AND CAPACITY

If the contents of a 1 ℓ bottle are poured into a cube-shaped container with internal measurements of 10 cm × 10 cm × 10 cm, it will fill the container exactly. Thus:



$$
\begin{aligned}
&(10\text{ cm} \times 10\text{ cm} \times 10\text{ cm}) &= \ &1\ \ell\\
\text{or}\quad &1\,000\text{ cm}^3 &= \ &1\ \ell
\end{aligned}
$$

$$
\begin{aligned}
\text{Since}\quad & 1\ \ell &= \ & 1\,000\text{ ml} & \\
& 1\,000\text{ cm}^3 &= \ & 1\,000\text{ ml} & [1\ \ell = 1\,000\text{ cm}^3]\\
\therefore\quad & 1\text{ cm}^3 &= \ & 1\text{ ml} & [\text{divide both sides by } 1\,000]
\end{aligned}
$$

$$
\begin{aligned}
\text{Since}\quad & 1\text{ kl} &= \ & 1\,000\ \ell & \\
& &= \ & 1\,000 \times (1\,000\text{ cm}^3) & [1\ \ell = 1\,000\text{ cm}^3]\\
& &= \ & 1\,000\,000\text{ cm}^3 & \\
& &= \ & 1\text{ m}^3 & [1\,000\,000\text{ cm}^3 = 1\text{ m}^3]
\end{aligned}
$$

This means that an object with a volume of 1 cm³ will take up the same amount of space as 1 ml of water. Or an object with a volume of 1 m³ will take up the space of 1 kl of water.

The following diagram shows the conversions in another way:

$$
\begin{aligned}
&1\text{ ml} \xrightarrow{\times 1\,000} 1\ \ell\ (\text{or } 1\,000\text{ ml}) \xrightarrow{\times 1\,000} 1\text{ kl}\ (\text{or } 1\,000\ \ell)\\
&1\text{ cm}^3 \xrightarrow{\times 1\,000} 1\,000\text{ cm}^3 \xrightarrow{\times 1\,000} 1\text{ m}^3\ (\text{or } 1\,000\,000\text{ cm}^3)
\end{aligned}
$$

Conversion is the changing of something into something else. In this case, it refers to changes between equivalent units of measurement.

From the diagram on the previous page, you can see that:

- 1 ℓ = 1 000 ml; 1 ml = 0,001 ℓ
- 1 kl = 1 000 ℓ; 1 ℓ = 0,001 kl
- 1 ml = 1 cm^3
- 1 ℓ = 1 000 cm^3
- 1 kl = 1 000 000 cm^3 or 1 m^3

Remember these conversions:
1 ml = 1 cm^3
1 kl = 1 m^3

VOLUME AND CAPACITY CALCULATIONS

1. Write the following volumes in ml:

   (a) 2 000 cm^3 .... (b) 250 cm^3 ....
   (c) 1 ℓ .... (d) 4 ℓ ....
   (e) 2,5 ℓ .... (f) 6,85 ℓ ....
   (g) 0,5 ℓ .... (h) 0,5 cm^3 ....

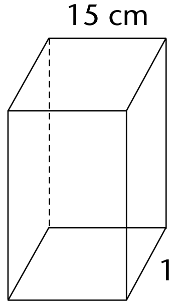
2. Write the following volumes in kl:

   (a) 2 000 ℓ .... (b) 2 500 ℓ ....
   (c) 5 m^3 .... (d) 6 500 m^3 ....
   (e) 3 000 000 cm^3 .... (f) 1 423 000 cm^3 ....
   (g) 20 ℓ .... (h) 2,5 ℓ ....

3. A glass can hold up to 250 ml of water. What is the capacity of the glass:

   (a) in ml? .... (b) in cm^3? ....

4. A vase is shaped like a rectangular prism. Its inside measurements are 15 cm × 10 cm × 20 cm. What is the capacity of the vase (in ml)?

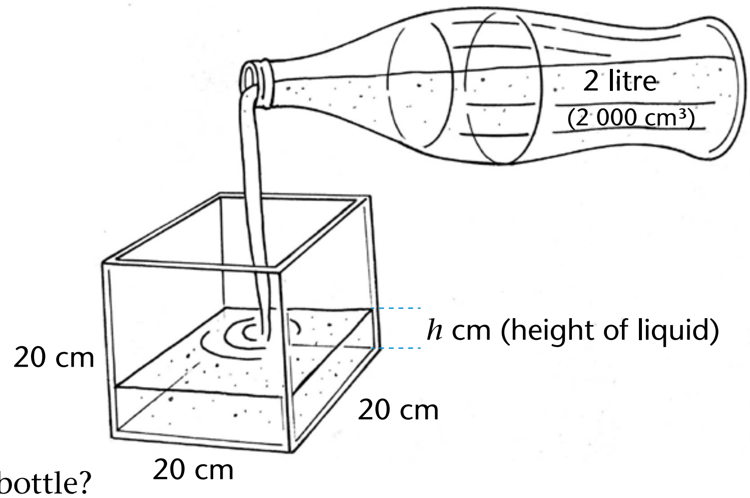


....

....

....

5. A liquid is poured from a full 2 ℓ bottle into a glass tank with inside measurements of 20 cm by 20 cm by 20 cm.



(a) What is the volume of the liquid when it is in the bottle?

....

(b) What is the capacity of the bottle?

....

(c) What is the volume of the liquid after it is poured into the tank?

....

(d) What is the capacity of the tank?

....

....

(e) How high does the liquid go in the tank?

....

....

In question 5 above, you should have found the following:

$$\begin{aligned}\text{Volume of liquid in tank} &= \text{Volume of liquid in bottle}\\ 20 \times 20 \times h \text{ (liquid's height in tank)} &= 2\,000 \text{ cm}^3\\ h &= \frac{2000}{(20 \times 20)}\\ &= 5 \text{ cm}\end{aligned}$$

Note: The capacity of the tank is 20 cm × 20 cm × 20 cm = 8 000 cm³ (8 ℓ). The volume of liquid in the bottle is 2 000 cm³ (2 ℓ).

WORKSHEET

1. Do the following unit conversions:

   (a) 2 348 cm^2 = ... m^2

   (b) 5,104 m^2 = ... cm^2

   (c) 1 m^3 = ... kl

   (d) 250 cm^3 = ... ml = ... ℓ

   (e) 0,5 kl = ... ℓ = ... ml

   (f) 6,850 ℓ = ... ml = ... cm^3

2. A rectangular prism measures 8 m × 4 m × 3 m. Calculate:

   (a) its surface area

   (b) its volume

   ....

   ....

   ....

3. A boy has 27 cubes, with edges of 20 mm. He uses these cubes to build one big cube.

   (a) What is the volume of the cube if he uses all 27 small cubes?

   ....

   ....

   (b) What is the edge length of the big cube?

   ....

   (c) What is the surface area of the big cube?

   ....

   ....

4. A glass tank has the following inside measurements: length = 250 mm, breadth = 120 mm and height = 100 mm. Calculate the capacity of the tank:

   (a) in cubic centimetres

   ....

   ....

(b) in millilitres

....

(c) in litres

....

5. Calculate the capacity of each of the following rectangular containers. The inside measurements have been given.

	Length	Breadth	Height	Capacity
(a)	15 mm	8 mm	5 mm	..............cm^3
(b)	2 m	50 cm	30 cm	..............ℓ
(c)	3 m	2 m	1,5 m	..............kl

....

....

....

6. A water tank has a square base with internal edge lengths of 150 mm. What is the height of the tank when the maximum capacity of the tank is 11 250 cm^3?

....

....

....

....

Term 2
Revision and assessment

Revision

You should not use a calculator for any of the questions in this chapter, unless you are told to use one. Do show all your steps of working.

FRACTIONS

1. Calculate the following:

   (a) $3\frac{2}{5} + 2\frac{4}{5}$

   ....

   ....

   (b) $4\frac{1}{3} - 3\frac{5}{6}$

   ....

   ....

   (c) $\frac{13}{4} - \frac{4}{3}$ (give your answer as a mixed number)

   ....

   ....

   (d) $2\frac{1}{3} \times 1\frac{1}{14}$

   ....

   ....

2. Three quarters of a number is 63. What is the number?

   ....

3. Write down all the fractions in this list that are smaller than one eighth:

   $\frac{2}{8}$; $\frac{1}{7}$; $\frac{1}{9}$; $\frac{2}{17}$

   ....

4. The Stone Hill Primary U13A soccer team had a good season, winning five sixths of its matches. If the team played 12 matches that season, how many were *lost*?

   ....

   ....

5. For each sequence below, write down whether it is increasing, decreasing, or neither:

   (a) $\frac{1}{3}; \frac{1}{4}; \frac{1}{5}$ ....

   (b) $\frac{1}{3}; \frac{2}{6}; \frac{3}{9}$ ....

   (c) $\frac{1}{6}; \frac{2}{7}; \frac{3}{8}$ ....

   (d) $\frac{4}{3}; \frac{5}{4}; \frac{6}{5}$ ....

6. In a survey of 80 Grade 7 learners, 60% felt that Justin Bieber was the best singer. How many learners think he is the best singer?

   ....

7. Moeketsi collected R450 of the total of R3 000 collected by his class for the ABC for Life charity. What percentage of the total did Moeketsi collect?

   ....

   ....

8. BestWear had a sale on all its dresses. What was the percentage reduction on a dress that used to cost R600, but on sale was going for R480?

   ....

   ....

THE DECIMAL NOTATION FOR FRACTIONS

1. Re-order the following numbers from smallest to largest:

   (a) 0,04; $\frac{4}{10}$; 14%; 0,4%

   ....

   (b) 0,798; 0,789; 0,8; 0,79

   ....

2. What is the value of the 7 in 4,5678? Write your answer as a common fraction.

   ....

3. Fill in the missing numbers in the boxes below.

   (a) 7,99 $\boxed{}$ 8 (b) 9,123; 9,121; $\boxed{}$; 9,117; ...

4. Join all the pairs of numbers that *multiply together to give 1*. The first has been done for you. Note that you will not use all the numbers on the right-hand side.

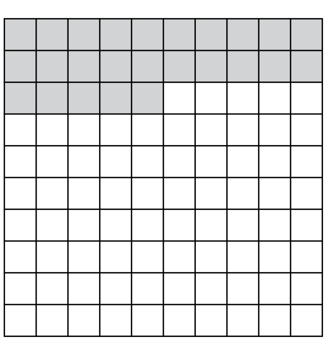
	0,5
1	10
0,2	1
0,5	5
0,02	2
0,1	50
	0,2

5. Calculate the following:

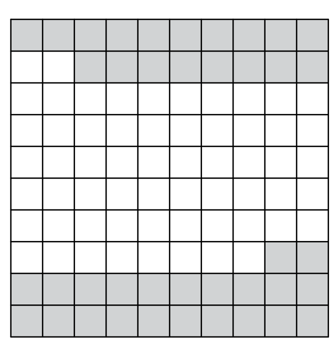
   (a) 5,673 – 3,597 (b) 4,85 × 1,2 (c) 4,825 ÷ 5

6. A certain portion of the shapes below are shaded. Write each portion as a common fraction (in simplest form), decimal fraction and percentage.

   (a)



   (b)



RELATIONSHIPS BETWEEN VARIABLES

1. (a) Here is a number sequence: 1; 4; 10; 22; ..... ; .....

   The rule for creating the number sequence is "times 2, add 2". Write down the next two numbers in the number sequence.

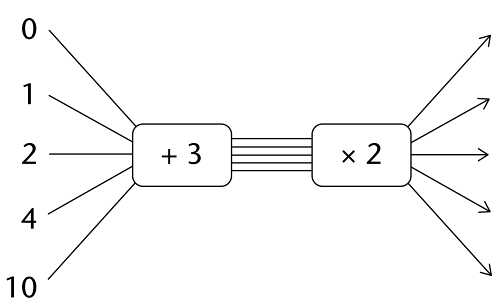
   (b) Here is another number sequence: 100; 50; 25; ...

   Write down, in words, the rule for creating this number sequence.

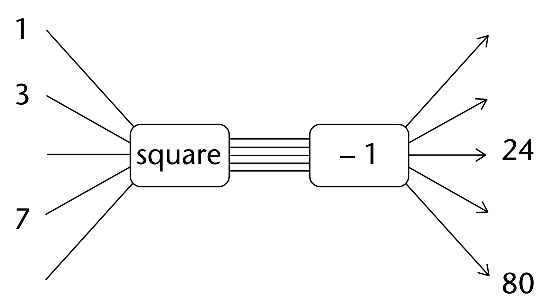
....

2. Use the given rule to calculate the missing values and/or determine the rule.

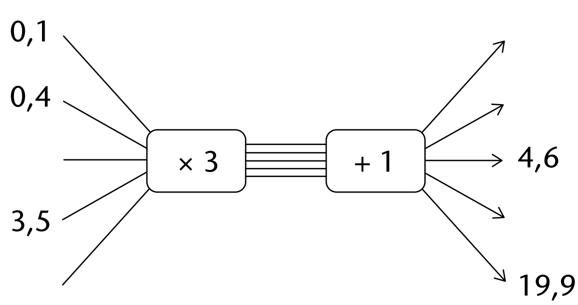
(a)



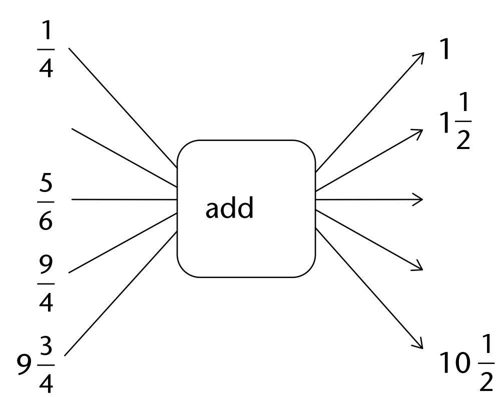
(b)



(c)



(d)



3. (a) There is a simple relationship (multiply by ...) between the y values and the x values in the table. Find it and then fill in the missing values.

x	0,1	0,3	0,6	2,5	3,2	
y	4	12	24			164

   (b) Write in words the rule that describes the relationship between the x values and the y values.

....

4. (a) There is a simple relationship (add ...) between the x values and the y values in the table. Find the relationship and then fill in the missing values.

x	$\frac{1}{3}$	$\frac{5}{3}$	$\frac{7}{3}$	$9\frac{2}{3}$	$\frac{5}{6}$		$\frac{3}{4}$
y	$\frac{2}{3}$	2	$\frac{8}{3}$			15	$\frac{13}{12}$

(b) Write in words the rule by which the missing x and y values can be calculated.

....

5. The rule used to describe the relationship between the x values and y values in the table is "double the x and then subtract 2". Use the rule to find the missing values and fill them in.

x	4	8	12	15		
y	6	14			42	100

PERIMETER AND AREA OF 2D SHAPES

1. (a) A rectangle has an area of 48 cm^2 and a length of 8 cm. How wide is it?

....

....

(b) A different rectangle has an area of 72 cm^2, and is twice as long as it is wide. Determine the dimensions of this rectangle.

....

....

....

(c) A triangle has a base of 10 cm and an area of 20 cm^2. What is the height of the triangle?

....

....

....

(d) What is the length of the side of a square that has an area of 144 cm^2?

....

....

2. An equilateral triangle with sides of 8,4 cm and a square have the same perimeter. Determine the length of the side of the square.

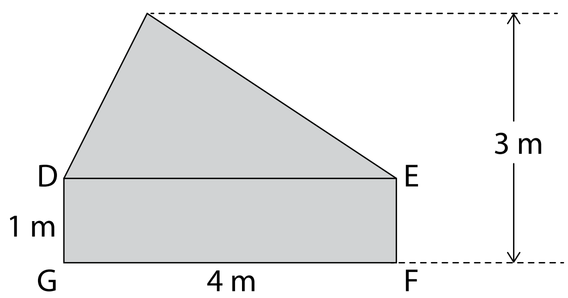
....

....

....

3. Calculate the area of the shaded figures.

(a) DEFG is a rectangle. Dimensions of the sides are as indicated.



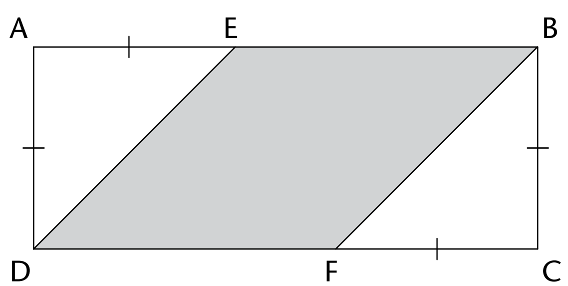
....

....

....

....

(b) ABCD is a rectangle. AB = 5 cm and FC = 2 cm. Give your answer in square millimetres. (You may use a calculator in this question.)



....

....

....

....

....

....

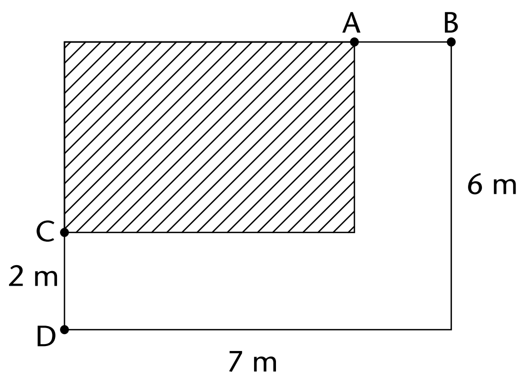
....

....

....

4. The garden of Mr and Mrs Mbuli is shown below, *not* to scale. There is a hedge all around the garden, except for the 2 metre wide gate (from A to B). The shaded area is grassed (the rest has trees, shrubs etc.).

   Garden Dream quoted the Mbulis R5 per square metre to mow their lawn and R10 per metre to trim their hedge. VAT is included in these prices. What was the total amount that Garden Dream quoted?



....

....

....

....

....

....

....

....

SURFACE AREA AND VOLUME OF 3D OBJECTS

1. How many litres of water will a fish tank with inside measurements of 1,2 m × 60 cm × 70 cm hold, if it is filled to the brim?

   ....

   ....

2. A rectangular prism has a length of 4 cm, a width of 10 cm and a volume of 240 cm^3. What is the height of the prism?

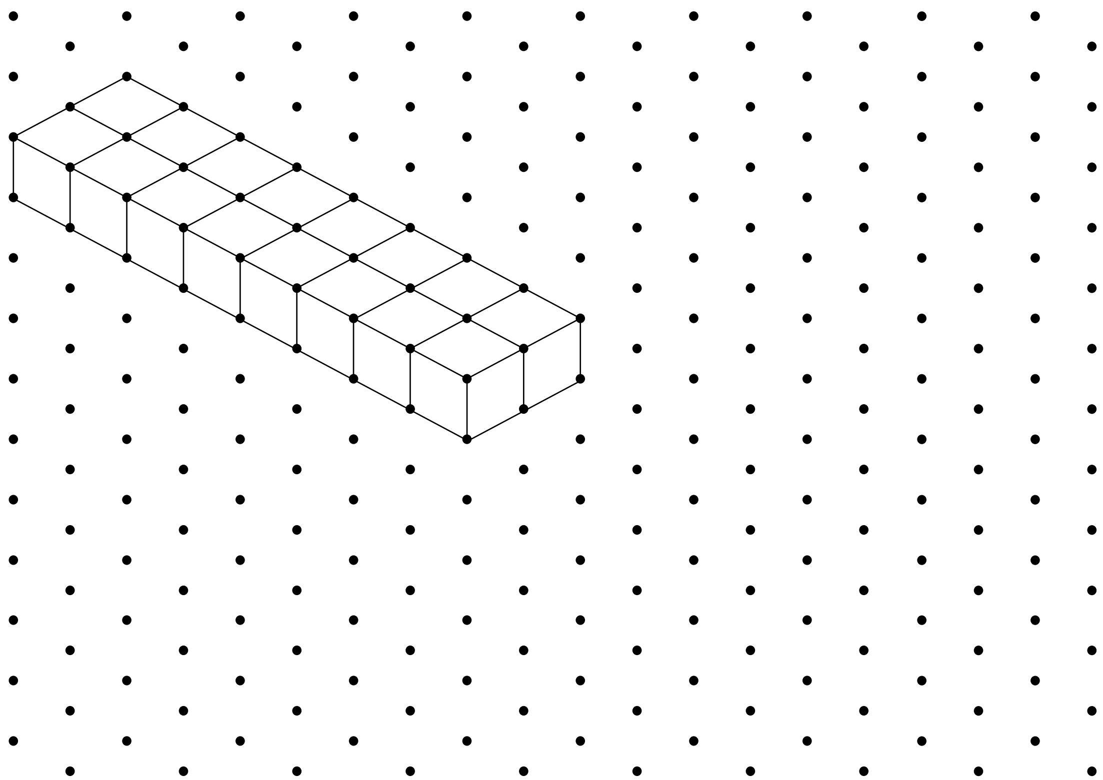
   ....

   ....

3. A rectangular prism has a certain volume. Which of the following will double the volume of the prism? Tick the correct answer(s).

   - ☐ Doubling all the dimensions
   - ☐ Doubling the length only
   - ☐ Doubling the length and the width, and halving the height
   - ☐ Doubling the length and halving the width and keeping the height unchanged

4. Look at the diagram below of a rectangular prism made out of 16 cubes.



Draw on the same grid two different rectangular prisms with the same volume as the one shown.

5. The total surface area of a cube is 150 cm^2. Determine the volume of the cube.

....

....

....

....

6. The volume of a cube is 64 cm^3. Determine the total surface area of the cube.

....

....

....

7. In order to save water when flushing the toilet, Mrs Patel added a solid brick to the cistern. The internal dimensions of the cistern are 30 cm × 30 cm × 10 cm, and the brick together with other internal mechanisms have a volume of 1 000 cm^3.

   (a) Calculate how many litres of water the cistern holds if the water fills up to 5 cm below the top of the cistern.

   ....

   ....

   ....

   (b) Suppose the Patel family flush the toilet an average of 12 times a day. Use your calculator to determine how many kilolitres of water they will use by this means in one year.

   ....

8. Njabulo wishes to varnish the outside of a wooden chest that is in the shape of a rectangular prism. The bottom of the chest does not need to be varnished as it is on the ground. The chest is 1,5 m long, 50 cm wide and 80 cm high. Determine, in square metres, the total surface area that will need to be varnished.

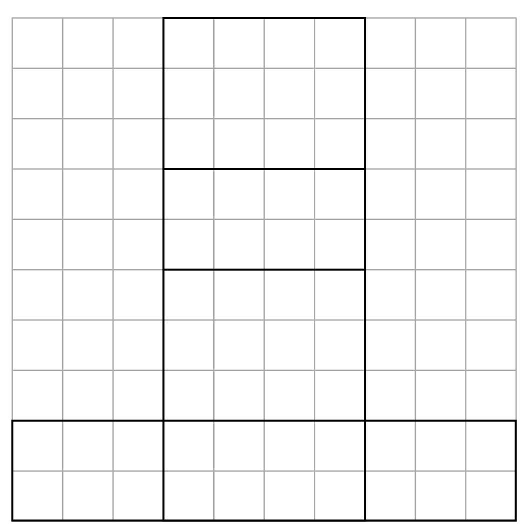
   ....

   ....

   ....

   ....

9. The image on the right shows the net of a rectangular prism drawn on a grid. If each block on the grid is a square with a side length of 1 unit, calculate:



   (a) The total surface area of the prism

   ....

   ....

   (b) The volume of the prism

   ....

   ....

Assessment

In this section, the numbers indicated in brackets at the end of a question indicate the number of marks that the question is worth. Use this information to help you determine how much working is needed.

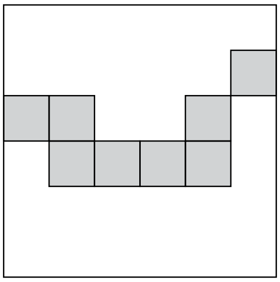
The total number of marks allocated to the assessment is 60.

Note:
Do not use your calculator!

1. $\frac{1}{4}$ is half of x. What is the value of x? (2)

   ....

2. The diagram alongside shows a square made up of blocks. Eight of these blocks have been shaded. Write, in its simplest form, the fraction of the square that is shaded. (2)



   ....

3. Calculate the following:

   (a) $2\frac{2}{3} \times 1\frac{1}{2}$ (b) $\frac{13}{10} - \frac{5}{8}$ (6)

   ....

   ....

4. Mrs Baker has baked a cake. She has some ladies around for tea and they eat half the cake. Her son John eats a quarter of the rest of the cake. What fraction of the cake is left? (2)

   ....

   ....

5. The price of petrol has risen from R8 per litre to R12 per litre over the past 2 years. Determine the percentage increase in the price. (2)

   ....

6. The Cupidos moved home. In the move, 5% of their crockery got broken. They have 57 pieces of crockery left (unbroken). How many pieces broke in the move? (2)

....

....

....

....

7. $\frac{15}{400} = 0{,}0375$; $\frac{17}{400} = 0{,}0425$; $\frac{19}{400} = 0{,}0475$

Using the above information, write down the decimal equivalents of the following fractions:

(a) $\frac{21}{400}$ = ...

(b) $\frac{22}{400}$ = ...

(c) $\frac{13}{400}$ = ... (3)

8. Multiply 56,76147 by 100 and round off your answer to two decimal places. (2)

....

9. Buti goes to the store and buys two cooldrinks at R7,50 each and three packets of chips at R5,95 each. If he pays with a R50 note, how much change should he get? (4)

....

....

....

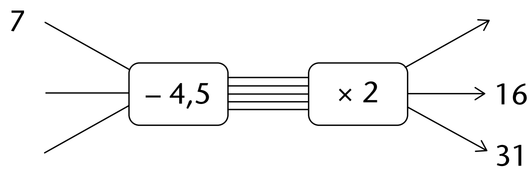
....

10. Class 7A at Grace Primary School collects some money for 3 charities. If the total they collect is R823,80, and the money is allocated equally to each charity, how much will each charity receive? (2)

....

....

11. Use the given rule to calculate the missing values: (3)

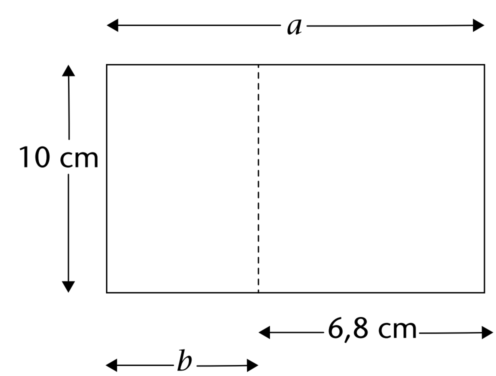


(a) There is a simple relationship (add ...) between the values of x and those of y. Find the relationship and then write down the missing values into the table. (2)

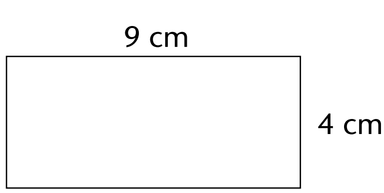
x	0,15	0,76	0,99	1,71	
y	1,4	2,01	2,24		18,93

(b) Write in words the rule by which the missing x and y values can be calculated. (1)

12. The total area of the rectangle shown is 112 cm². Determine the lengths of a and b. (3)



13. Below is a rectangle, with dimensions as shown. A square has the same perimeter as the rectangle below. How long is the side of the square? (2)



14. The diagram shows a rectangle divided into a triangle and a trapezium. Calculate the shaded area, giving your answer in mm². (5)

....

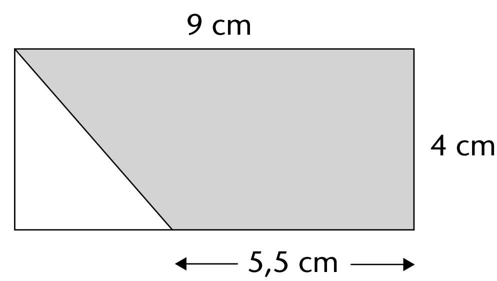
....

....

....

....

....



15. The length and width of a rectangle is doubled.

(a) Tick the statement that is correct:

☐ The perimeter of the rectangle stays the same.

☐ The perimeter of the rectangle doubles.

☐ The perimeter of the rectangle increases but it is not possible to say exactly by how much.

(b) Tick the statement that is correct:

☐ The area of the rectangle stays the same.

☐ The area of the rectangle doubles.

☐ The area of the rectangle triples.

☐ The area of the rectangle increases to 4 times what it was before.

(c) Explain your answer to part (b). (3)

....

16. A rectangular prism has a volume of 24 cm³. In the table below, write four possible dimensions that the prism may have. One possible combination has already been added. Note: do not consider, for example, a prism with length 6 cm, and height and width 2 cm to be different. (4)

Length	**Width**	**Height**
2 cm	2 cm	6 cm

17. The inside of the boot of a car is in the shape of a rectangular prism, with length 1,2 m, width 70 cm and depth 40 cm. Determine the capacity of the boot *in litres*. (3)

....

....

....

18. The volume of a cube is 27 cm^3. Determine the surface area of the cube. (3)

....

....

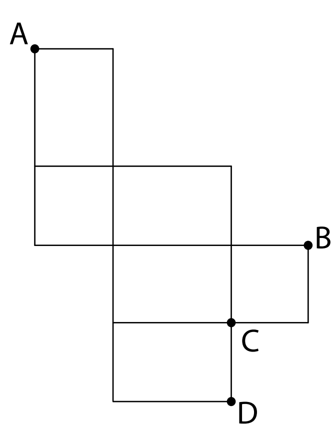
....

19. The length and breadth of a rectangular prism are both 4 cm, and its volume is 48 cm^3. Determine the height of the prism. (2)

....

....

20. Consider this net:



(a) What is the name of the solid created if this net is folded?

....

(b) Which corner will A touch when the solid is created: B, C or D? (2)

....

Notes to myself

Notes to myself

Notes to myself